Transparency on the Consumer Side and Tacit Collusion

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Abstract

This paper investigates the effects on tacit collusion of increased market transparency on the consumer side of a market in a differentiated Hotelling duopoly. Increasing market transparency increases the benefits to a firm from undercutting the collusive price. It also decreases the punishment profit. The net effect is that collusion becomes harder to sustain. In the limiting homogeneous market, the effect vanishes. Here market transparency does not affect the possibilities for tacit collusion.

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1 Introduction

In the competition policy debate, improved transparency is typically viewed as promoting competition if it affects the consumer side of the market. The arguments usually refer to a static setting, while the important anti-trust considerations often concern tacit collusion. In contrast, improved transparency on the producer side is typically thought to be anti-competitive, see for instance Kühn and Vives (1995). If firms are uncertain about their competitors' prices, tacit collusion is harder to maintain, as price undercutting is harder to detect. Taken alone this effect implies that increased transparency facilitates collusion. This effect is well understood and we will not concentrate on it. Improved transparency on the consumer side has countervailing effects. In a static setting, the market becomes more competitive as the effective demand elasticity of a firm increases. However, the effects on collusion are ambiguous. The increased elasticity of demand makes it more tempting to undercut the other firm; this destabilizes collusion. On the other hand, a more severe punishment is possible in a transparent market. This facilitates collusion. The total effect on collusion is the net effect of these two forces. We show that when goods are sufficiently differentiated, the first effect dominates, so improving transparency on the consumer side makes tacit collusion more difficult to sustain. In almost homogeneous markets, the effect vanishes and changes in transparency have almost no effect on collusion.

The degree of product differentiation is therefore important when one seeks to assess the competitive effects of improving market transparency. Granting that the effects on the producer side are anti-competitive, the net result of improving market transparency in a differentiated market is uncertain. Producer side and consumer side effects counter each other. However, in more homogeneous markets the consumer side effects vanish and focus should be on the producer side effects.

There has been renewed policy interest in transparency due to the advent of the Internet. While consumers before had to spend considerable time searching many markets, there are now many well-established web sites where price comparisons are available with a click on the mouse. Consumer agencies try to make markets more transparent in various ways, though magazines, press releases, and also through websites. Newspapers often have weekend sections, where a market is surveyed and prices compared. In these (and many other) cases market transparency is improved not by the firms selling in the market but by outside agents.

We identify market transparency on the consumer side with consumers' information about the prices charged by the firms in the market. Some consumers are informed about prices and some are not. Informed consumers are supposed to know both firms' prices, as they would if they had access to a web site comparing prices. This follows the "Sales Model" of Varian (1980). The uninformed consumers have an expectation about the firm's price. In equilibrium, this expectation is correct. Nevertheless, the uninformed consumers affect the amount of demand a firm can gain by lowering its price - and therefore the effective demand elasticity - as they do not learn about the price change.

Varian shows that when the market is homogeneous and some consumers are uninformed about price, the static Nash equilibrium is in mixed strategies. This feature is present in our model when product differentiation is low[1], but not when it is high.

Tacit collusion requires that a potential deviation from the collusive path is discouraged by a sufficiently hard punishment. As is well known, the credibility of very hard punishments can be questioned. In this paper we

[1] Actually, the precise condition for when the Nash equilibrium is in mixed strategies also depend non-monotonically on the degree of transparency, so a low degree of product differentiation means low relatively to the level of market transparency.

study the case where the punishment consists of reversion to the static Nash equilibrium in all future. The working paper version also briefly studies the case of optimal symmetric punishments for some parts of the parameter space, and the results are qualitatively similar.

Increasing transparency makes collusion more difficult to sustain in a differentiated market. This shows up in two ways: first, the smallest discount factor necessary for sustaining collusion on the monopoly price is increasing in market transparency. Secondly, for lower discount factors, the best price the firms can collude on (if any) is decreasing in the level of market transparency.

In the limiting case of a homogeneous market, the crucial discount factor is independent of market transparency, since deviation gains and punishment are affected equally. In this sense, the homogeneous and differentiated markets are qualitatively different, but it is also shown that the effect in the differentiated market tends to zero as product differentiation disappears. Furthermore, it is shown that in the homogeneous market collusion is impossible if the discount factor is too low for collusion at the monopoly price. The same feature is true when product differentiation is low. Also in this respect, the homogeneous market is the limit of the differentiated.

Market transparency is viewed differently in different countries and by different competition authorities. While the previous Danish competition act actively tried to promote transparency in order to increase competitiveness (see Albæk, Møllgaard and Overgaard (1994)), the Danish government now acknowledges that, although transparency may be good in so far as it increases consumers' information, it may help firms collude if it increases firms' information, see e.g. Erhvervspolitiken (1999). The EU commission seems to have a mixed view. Kuhn and Vives (1995) conclude that the Commission mostly found increased transparency in the form of price-announcements by firms as anti-competitive. However, the internal European market as well as the single European currency has often been seen as adding to transparency and therefore competitiveness.

Market transparency has been analyzed from different angles in the literature. As discussed above, Varian (1980) studied a homogeneous market, where market transparency is incomplete. He showed that the expected profit of the firms in the symmetric Nash equilibrium decreases in the level of market transparency. In this sense increasing market transparency intensifies competition. The search literature, see for instance Burdett and Judd (1993) or Stahl (1989) can be seen as a developing this theme. Lowering search costs increases search and this intensifies competition. Anderson and Renault (1999) study price competition when consumers have to search for prices and product characteristics. They show that market prices rise with search costs. The literature on advertising, see e.g. Bester and Petrakis (1995), can also be seen as contributing to understanding market transparency. An important difference, however, is that in this case the firms in the market actively affect market transparency, while we consider the case where outside agents affect market transparency. Increased advertising is typically shown to lead to lower prices.

On the other hand, cartel theory stemming back to Stigler (1964), Green-Porter (1984) and others, see Tirole (1988) for an overview, has pointed to the anti-competitive effects of increased transparency on the producer side of the market for the reasons described above.

There has been little work on the effects consumer side transparency on collusion. An exception is Nilsson (1999). He considers tacit collusion in a repeated search model of a *homogeneous* market a la Burdett and Judd (1993), where firms are perfectly informed of prices but consumers have to search in order to learn prices. Lowering search cost corresponds to increasing transparency in Varian's model; it lowers the expected price in a one shot equilibrium. Nevertheless collusion is facilitated in the repeated game. In Nilsson's model, most of the consumers decide whether to search or not taking into account the expected benefits from searching, while a fraction of the consumers always search. The majority of consumers therefore stop searching if firms set the same price. This happens if the firms collude on a high price. Undercutting the other firm in the collusive phase will therefore only give a relatively small increase in demand. This feature facilitates collusion. In the punishment phase of the collusive equilibrium, firms do not set the same price (they play a mixed strategy as in Varian), therefore search occurs and increasing transparency through lowering search costs intensifies competition in this phase. Thus increasing transparency increases search in the punishment phase but not in the collusive phase, as a result increasing transparency facilitates collusion.

In the present paper, on the contrary, increasing transparency increases the information level of consumers in both phases of the equilibrium and the result therefore depends on the net effect on deviation profit and punishment profit. The other important difference is that we consider both a homogeneous and a *differentiated* market.

Møllgaard and Overgaard (2000) study a repeated differentiated duopoly, but do not distinguish between substitutability and transparency. They assume that products really are homogeneous, but that consumers are not rational and *believe* that goods are differentiated. The degree of substitutability of the goods in the consumers' utility functions is interpreted as *perceived* substitutability and identified with transparency. Their results therefore reflect the results of the literature on stability of collusion in differentiated markets. See, for example, Deneckere (1983), Chang (1991), Ross (1992) and Häckner (1995). These authors show that as products become more substitutable, a deviation becomes increasingly attractive. The punishment phase also becomes more severe. Sustainability of tacit collusion is the net effect of these two forces. Deneckere (1983) shows that with price setting firms and Nash reversion the relation is non-monotonic and that for fairly homogeneous goods collusion gets more difficult with greater product differentiation. Møllgaard and Overgaard suggest that this implies that limiting market transparency could reduce the scope for collusion when markets are very transparent. Our result shows that this conclusion is not valid if one models transparency explicitly.

The organization of the paper is the following. Section 2 introduces a simple Hotelling market. The one shot Nash equilibrium is characterized in Section 3 for the different cases of product differentiation. Section 4 studies the effect of transparency on tacit collusion, while Section 5 concludes.

2 The market

Consider a Hotelling market with a continuum of consumers. Consumer x is located at $x \in [0,1]$. Each consumer either buys one unit of the (differentiated) good or does not buy. There are 2 firms, located at 0 and 1 respectively. If firms charge prices p_0 and p_1, consumer x gets utility $u - p_0 - tx$ from buying one unit from firm 0 and $u - p_1 - t(1-x)$ from buying a unit from firm 1. The degree of product differentiation is given by the parameter $t > 0$. We will assume that all consumers are potential costumers at each firm: $u \geq t$. A consumer, who is informed about the prices of the firms, is indifferent between buying from 0 and 1 if she is located at

$$x(p_0, p_1) \equiv \frac{1}{2} + \frac{p_1 - p_0}{2t}. \tag{1}$$

There are two different *information types* of consumers: a fraction ϕ are informed about both firms' prices, while a fraction $(1-\phi)$ are uninformed. The variable ϕ is our measure of market transparency, the higher is ϕ, the more transparent is the market. We conceive of the informed consumers as having easy access to the pricing information, perhaps through an internet site. Both information types of consumers are supposed to be uniformly distributed on locations. In principle one could of course also imagine that some consumers are only informed about one of the firms' price. We will however leave these complications aside. We assume that

$$\frac{t}{u} < \frac{2\phi}{2+\phi}; \tag{2}$$

this will imply that the market is covered.

All consumers know the locations of the firms, regardless of whether they are informed about the firms' prices or not. A consumer, who is uninformed about firm $i's$ price has an *expectation* p_i^e of this price. For this consumer, the *expected* utility from buying from firm i is $u - p_i^e - tx$. An uninformed consumer is indifferent between buying from the two firms, if she is located at $x(p_0^e, p_1^e)$. A consumer can only visit one firm in a period; it is not possible for uninformed consumers to learn prices by visiting the firms in a sequence. The time line is as follows. First consumers form expectations, second firms set prices, which are observed by some consumers only. Based either on their knowledge or expectations about prices consumers decide on which firm to go to - if any. If an uninformed consumer arrives at a firm and finds that the price is higher than expected, the consumer may decline to buy if she wishes. This occurs, for instance, if she visits firm 0 and $u - tx - p_0 < 0$. In equilibrium this will not happen, as consumers have correct - rational - expectations. Finally, transactions take place.

If $p_0 - p_1 > t$, all informed consumers buy from firm 1, if $p_1 - p_0 > t$, they all buy from firm 0. In the sequel we will only consider symmetric equilibria where the expected prices are the same for the two firms, i.e., where $p_0^e = p_1^e = p^e$. As will become clear, the equilibrium price will be so low (less than or equal to $u - \frac{t}{2}$) that the market is covered and each firm faces $(1-\phi)/2$ uninformed consumers. The demand facing firm 0 can then be written without explicit reference to the expected prices as

$$D(p_0, p_1, \phi) = \begin{cases} \phi + \frac{1-\phi}{2} \text{ if } p_0 < p_1 - t \\ \phi\left(\frac{1}{2} + \frac{p_1 - p_0}{2t}\right) + \frac{1-\phi}{2} \text{ if } p_1 - t \leq p_0 \leq p_1 + t \\ \frac{1-\phi}{2} \text{ if } p_1 + t \leq p_0 \leq u - \frac{t}{2} \\ \frac{1-\phi}{2}\left(\frac{u-p}{t}\right) \text{ if } p_1 = u - \frac{t}{2} \leq p_0 \leq u. \end{cases} \tag{3}$$

For simplicity, we assume that marginal costs are constant. We normalize marginal costs to zero, so firm $0's$ profit in a period is $\pi_0 = p_0 D(p_0, p_1 \phi)$.

3 Static Nash equilibrium

We first concentrate on the one period Nash equilibrium. Depending on the degree of product differentiation relative to the maximal willingness to pay, t/u, and the transparency of the market, ϕ, the equilibrium will either be in pure or in mixed strategies.

3.1 Pure strategy Nash equilibrium

In a symmetric pure strategy equilibrium, the firms set the same price, serve both informed and uninformed consumers, and the relevant part of the demand function is given by the second line in (3). Firm 0's problem is

$$\text{Given } p_1, \phi \max_{p_0} \; p_0 D(p_0, p_1, \phi).$$

The best reply is

$$p_0 = \frac{1}{2}\left(p_1 + \frac{t}{\phi}\right).$$

The Nash equilibrium price $p^N(\phi)$ and profit, $\pi^N(\phi)$ are therefore given by

$$p^N(\phi) = \frac{t}{\phi}, \; \pi^N(\phi) = \frac{t}{2\phi}. \tag{4}$$

Both are clearly decreasing in ϕ; an increase in transparency increases competition and lowers the Nash-equilibrium price. When a firm decreases its price, only informed consumers notice it. The elasticity of demand therefore depends on transparency, it equals -$\frac{\phi}{2t}p$, when firms set the same price. An increase in transparency thus makes demand more elastic and competition more intense. In the one shot game the firms therefore - jointly $-$ have no interest in promoting market transparency. It is straightforward to check that the second order condition for maximum is fulfilled.[2]

It is clear from (4) (and the expression for the elasticity), that in the static, pure strategy, equilibrium, an increase in transparency is equivalent to a decrease in the degree of product differentiation. In this sense, introducing market transparency into the model just corresponds to a reparametrization of the degree of product substitutability.

3.2 Mixed strategy Nash equilibrium

When deriving the best reply in the last section, we assumed that it paid to serve some informed consumers (the second line in (3) was relevant). However, when goods are close substitutes the pure strategy Nash equilibrium price, $p = t/\phi$, becomes very low, and the equilibrium may not exist, since it may be better for a firm to raise its price and only sell to the $\frac{1-\phi}{2}$ uninformed consumers, who visit the firm expecting the low Nash equilibrium price (and then get a bad surprise).[3] If the firm decides to sell only to a fraction of the

[2]In deriving the equilibrium we assumed that the market is covered and the second line of (3) is relevant, hence it should not be advantageous to undercut the other firm by t and gain the whole informed market. This takes that $\left(\frac{t}{\phi} - t\right)\left(\phi + \frac{1-\phi}{2}\right) < \frac{t}{2\phi}$, which is fulfilled for all positive ϕ and t. Under assumption (2) the market is covered in the Nash equilibrium.

[3]One could object, that the ability of a firm to exploit the uninformed consumers rely on the fact that an uninformed consumer cannot visit the other firm if she is surprised by a high price. If the uninformed consumer could visit the other firm at a very low cost

uninformed consumers arriving, the best price solves $\max_{p_0} (1-\phi) \frac{u-p_0}{t} p_0$, if it decides to sell to all, $p_0 = u - \frac{t}{2}$. For small t, the best choice is $p_0 = u - \frac{t}{2}$. This gives higher profit than $p = \frac{t}{\phi}$ if

$$\frac{t}{u} < \frac{2(1-\phi)\phi}{(1+\phi)(2-\phi)}. \tag{5}$$

As

$$\frac{2(1-\phi)\phi}{(1+\phi)(2-\phi)} < \frac{2\phi}{2+\phi},$$

we see that (5) is a stronger restriction than (2). When (5) is fulfilled, the pure strategy Nash equilibrium derived above does not exist. Given the results of Varian (1980), this is not surprising. Varian showed that in a homogeneous market where a fraction ϕ of the consumers are uninformed, there are no pure strategy Nash equilibria, but a symmetric mixed strategy equilibrium exists. The same happens in our model, when the goods become close substitutes.[4] The crucial degree of substitutability depends non-monotonically on the degree of transparency. For high or low degrees of transparency, the Nash equilibrium will be in pure strategies even though product differentiation is low. This non-monotonicity stems from the fact that an increase in transparency both decreases the pure strategy Nash equilibrium profit and the profit the firm can obtain by raising the price and only serve uninformed consumers.

In the symmetric mixed strategy equilibrium both firms choose prices according to the distribution function F_t[5]. Let f_t be the corresponding density.

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if her expectations were proven wrong, then she could not be exploited as much, and the pure strategy Nash equlibrium would exist for lower t. However, when goods are almost homogeneous pure strategy Nash equilibrium will not exist even in this case as long a the cost of visiting the other firm is non-zero.

[4] As readers familiar with the literature will realize, the following is an adaptation of the analysis of Varian to our differentiated market.

[5] As we are amending Varian's (1980) analysis we just characterize the distribution

Since the equilibrium is symmetric, the uninformed consumers divide evenly between the two firms. As a firm will never set a price above $u - \frac{t}{2}$, all uninformed consumers buy regardless of the realization of the randomization[6]. If firm 0 chooses the price $p \leq u - \frac{t}{2}$ and firm 1 chooses prices according to F_t, firm $0's$ expected profit equals

$$E\pi_0 \equiv \left(\phi\left((1 - F_t(p+t)) + \int_{p-t}^{p+t}\left(\frac{1}{2} + \frac{p_1 - p}{2t}\right) f_t(p_1)dp_1\right) + \frac{1-\phi}{2}\right) p \tag{6}$$

If firm 1 sets a price, p_1, above $p + t$, firm 0 will sweep the whole demand from the informed consumers. This happens with probability $1 - F_t(p+t)$. If firm 1 sets a price in-between $p - t$ and $p + t$, demand from the informed consumers will be shared according to (1) and for prices p_1 below $p - t$, firm 0 receives no demand from the informed consumers. Furthermore, half of the uninformed buy from firm 0.

Integrating (6) by parts and simplifying yields

$$E\pi_0 = \left(\phi\left(1 - \frac{1}{2t}\int_{p-t}^{p+t} F_t(q)dq\right) + \frac{1-\phi}{2}\right) p \tag{7}$$

Let the supremum of the support of F_t be denoted $r(t)$, then $r(t) \leq u - \frac{t}{2}$. In a symmetric equilibrium both firms' mixed strategy is given by F_t. Since the expected profit is the same for all prices in the support of F_t, we can write the expected profit as

$$E\pi_0 = \left(\phi\Delta(t) + \frac{1-\phi}{2}\right) r(t) \tag{8}$$

function F_t directly and do not repeat some straightforward replications of Varian's lemmata (for instance that there are no atoms). F_t depends on t and ϕ, in order to save on notation, we only index F_t by t.

[6]Suppose, the firm sets a price $p > u - \frac{t}{2}$. Not all uninformed will buy and total demand will be $\frac{u-p}{t}$. The profit maximizing price is then $p = \frac{u}{2} < u - \frac{t}{2}$, so $p > u - \frac{t}{2}$ is not optimal.

Where

$$\Delta(t) \equiv \frac{1}{2} - \frac{1}{2t}\int_{r(t)-t}^{r(t)} F_t(q)dq \tag{9}$$

The terms $\phi\Delta\left(t\right)r\left(t\right)$ and $\frac{1-\phi}{2}r\left(t\right)$ reflect the rent the firm obtains from the informed and uninformed consumers respectively, when it charges $r\left(t\right)$. In the Appendix, we show that $r\left(t\right) \rightarrow u$ as $t \rightarrow 0$. Furthermore, using l'Hospital's rule we have that $\Delta\left(t\right) \rightarrow 0$ for $t \rightarrow 0$. We therefore have that

Lemma 1 *In the limit as* $t \rightarrow 0$, $E\pi_0 \rightarrow \frac{1-\phi}{2}u$

In the limiting homogeneous market, it is as if the firms are able to obtain all possible rent from the uninformed consumers, and none from the informed consumers, this was also shown by Varian. For the firms, static competition therefore works as if the uninformed consumers are exploited, while the informed - for whom the goods are perfect substitutes -retain all rent[7]. As the Lemma shows, this is also approximately true in the almost homogeneous market.

As is clear from (8) as well as Lemma 1, when the Nash equilibrium is in mixed strategies, market transparency does not just correspond to a reparametrization of product substitutability as it does when goods are fairly differentiated and the Nash equilibrium is in pure strategies. In the limiting homogeneous market profits depends on the monopoly (and not the competitive) price as well as the fraction of uninformed consumers. As we will see, a consequence is that tacit collusion is affected in different ways in these two cases.

[7]Since randomization takes place, the expected rent of the uninformed does not equal zero, but firms' expected profits are as if this were the case.

4 Tacit Collusion

Now we consider the repeated game. There are infinitely many periods, $\tau = 0, ..., \infty$. In each period the market is as described above. Firms seek to maximize the discounted sum of profits and both have the discount factor δ, which fulfills $0 < \delta < 1$. We will assume that a consumer's information type (as well as her location) is the same in all periods.

We focus on a trigger-strategy equilibrium, where in the collusive phase, firms collude on the best possible price, either the monopoly price, $p^m = u - \frac{t}{2}$ or some lower price. Deviations from collusion are punished with reversion to the one-shot Nash equilibrium for the rest of the game as suggested by Friedman (1971). We only consider the case, where (2) is fulfilled, i.e. where the products are reasonably close substitutes, so that the firms cannot sustain the monopoly price in a one shot game.

If the firms collude on the price p, each firm earns a profit $\pi(p) = \frac{p}{2}$ in all periods. The best deviation price maximizes the one period profit, $p'D(p', p, \phi)$. If a firm deviates and lowers its price, only a fraction ϕ will learn that the price is lowered before they visit the firm. The rest $1 - \phi$ expect the firm to set p. Half of these consumers will visit the firm and get a nice surprise and will not decline to buy. The other half will not observe the deviation, as they buy from the other firm. The optimal deviation price is given by

$$p^d = \begin{cases} \frac{1}{2}\left(p + \frac{t}{\phi}\right) \text{ if } p \leq 2t + \frac{t}{\phi} \\ p - t \text{ if } p > 2t + \frac{t}{\phi}. \end{cases} \tag{10}$$

Notice that $\frac{1}{2}\left(p + \frac{t}{\phi}\right)$ is decreasing in ϕ and less than the collusive price p, when $p > t/\phi = p^N$. The more transparent the market is, the more demand is captured by a price decrease, and the lower is the optimal price. The associated profit is

$$\pi^d(p) = \begin{cases} \frac{1}{8}\frac{(\phi p+t)^2}{\phi t} \text{ if } p \leq 2t + \frac{t}{\phi} \\ (p-t)\frac{1+\phi}{2} \text{ if } p > 2t + \frac{t}{\phi}. \end{cases} \tag{11}$$

Both expressions are increasing in ϕ when $p > t/\phi$. Hence, the more transparent the market, the more can potentially be gained from deviating from collusive play. Clearly, this effect by itself makes collusion harder to sustain.

A deviation is punished by reversion to the Nash equilibrium in all future. Let π^N denote the (expected) profit of the Nash equilibrium. Collusion on the price p can be sustained if the present value of monopoly profits exceeds the deviation profit plus the present value of the punishment profits, i.e., if

$$\frac{1}{1-\delta}\pi(p) \geq \pi^d(p) + \frac{\delta}{1-\delta}\pi^N. \tag{12}$$

If the firms collude on the monopoly price $p^m = u - \frac{t}{2}$, they each earn $\pi^m = p^m/2$ in each period. If $p^m > 2t + \frac{t}{\phi}$, or

$$\frac{t}{u} < \frac{2\phi}{5\phi+2}, \tag{13}$$

then the deviation profit is given by the second expression in (11) otherwise it is given by the first. Figure 1 below plots the conditions (2), (5), and (13).

4.1 Relatively low product differentiation

We first consider the case where product differentiation is relatively low and (5) is fulfilled so that the Nash equilibrium is in mixed strategies. We will focus on the case where product differentiation is so low, that the optimal deviation price is given by the second part of the expression (10). As is clear from Figure 1, this is fulfilled in most of the relevant parameter space.

Assume that the firms are able to collude on the monopoly price, $p^m = u - \frac{t}{2}$. Inserting the second part of (8) and (11) into the non-deviation

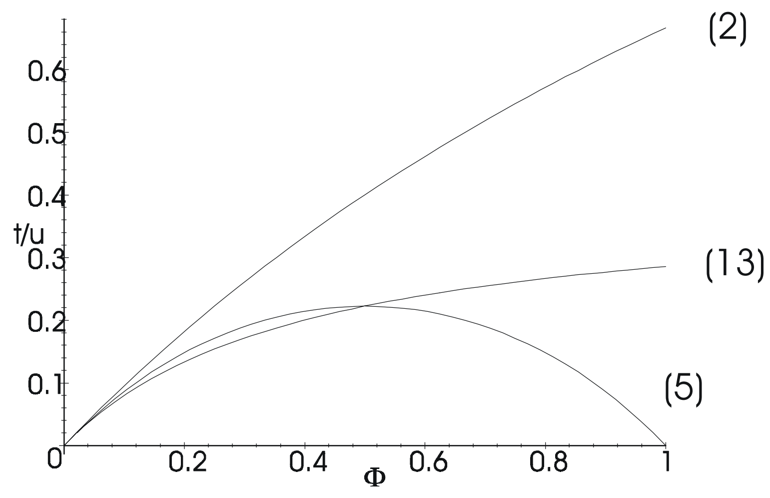

Figure 1:

constraint (12) gives

$$\frac{1}{1-\delta}\frac{p^m}{2}=(p^m-t)\frac{1+\phi}{2}+\frac{\delta}{1-\delta}E\pi_0$$

Solving for δ gives the lowest discount factor compatible with full collusion,

$$\delta_1(t,\phi)=\frac{\phi(p^m-t)-t}{(p^m-t)(1+\phi)-2E\pi_0} \tag{14}$$

In the limit, as $t\to 0$, we get, using Lemma 1,

$$\lim_{t\to 0}\delta_1(t,\phi)=\frac{u\phi}{u+u\phi-2\frac{1-\phi}{2}u}=\frac{1}{2}$$

This is *independent* of ϕ. In the homogeneous market, an increase in ϕ leads to an increase in the deviation profit which exactly balances the decrease in the profit of the punishment, so that the incentive constraint is unchanged.

In the differentiated market, we find the effect of increasing ϕ by differentiating $\delta_1(\phi,t)$ wrt ϕ. We get

$$\frac{\partial\delta_1(t,\phi)}{\partial\phi}=\frac{p^m-t}{p^m+p^m\phi-t-t\phi-2E\pi_0}-\frac{p^m\phi-t-t\phi}{(p^m+p^m\phi-t-t\phi-2E\pi_0)^2}\left(p^m-t-2\frac{\partial E\pi_0}{\partial\phi}\right) \tag{15}$$

Unfortunately, the sign of $\frac{\partial \delta_1(t,\phi)}{\partial \phi}$ cannot directly be assessed for all relevant t. Using Lemma 1, we get

$$\lim_{t\to 0} \frac{\partial \delta_1(t,\phi)}{\partial \phi} = \lim_{t\to 0} \frac{1}{4u\phi}\left(u + 2\frac{\partial E\pi_0}{\partial \phi}\right)$$

In the Appendix, we prove that $\lim_{t\to 0} \frac{\partial E\pi_0}{\partial \phi} = -\frac{u}{2}$. We thus have

Proposition 1. *Suppose that goods are almost homogeneous. Then there is almost no impact of market transparency on the scope for collusion. More formally, the lowest discount factor allowing full collusion on the monopoly price,* $\delta_1(t,\phi)$ *fulfills* $\lim_{t\to 0} \delta_1(t,\phi) = \frac{1}{2}$, *and* $\lim_{t\to 0} \frac{\partial \delta_1(t,\phi)}{\partial \phi} = 0$ *for all* ϕ.

When goods are almost homogeneous, firms' profits in the mixed Nash equilibrium are as if firms extract all rent from the uninformed consumers and none from the informed. It is like selling to the uninformed at the monopoly price. A change in market transparency does not change this, it only changes the share of uninformed agents. Colluding at the monopoly price means that the firms now also extract all rent from the informed consumers. If product differentiation is vanishingly low, a firm can gain the whole informed market by undercutting slightly, i.e. double the earnings from the informed. The punishment consists of loosing the rent from the informed only. As the monopoly price does not depend on market transparency, changes in market transparency does not change the involved profits per consumer, it only affects the size of the informed market. Therefore the result is the same as in a homogeneous perfectly informed market, the crucial discount factor is one half, reflecting that a deviation captures all rent in the informed part of market, the punishment consists of loosing half of all potential rent from this part of the market. The only difference is that in our setting these incentives only apply to the informed part of the market, but since this is true on both sides of the incentive constraint, changes in the relative size of the informed market do not matter for the result.

The Proposition only gives the limit result. As a closed form for the mixed strategy of the Nash equilibrium is not available, I have not been able obtain a result for larger t. I have carried out simulations for small t, the simulations rest on approximations for Δ and r and are therefore only indicative. In the simulations, $\frac{\partial \delta_1(t,\phi)}{\partial \phi}$ is positive (and of course small)[8]. This indicates that although the effect is small, it is qualitatively the same as in the more differentiated market.

Now, consider the case where the discount factor is so low, that the firms cannot collude on the monopoly price, $\delta < \delta_1(t,\phi)$. We consider the possibilities for collusion on prices $p \in [2t + \frac{t}{\phi}, u - \frac{t}{2}]$ so the second part of (11) is relevant. Inserting this and (8) in the non-deviation constraint (12) and solving for p yields

$$p_c^1 = \frac{\frac{\delta}{1-\delta} r(t) \left(\frac{1-\phi}{2} + \phi \Delta(t) \right) - t \frac{1+\phi}{2}}{\frac{1}{2(1-\delta)} - \frac{1+\phi}{2}} \qquad (16)$$

Consider first the homogeneous market (where $t = 0$, and $\Delta(t) = 0$, and $r(t) = u$). Here, p_c^1 reduces to

$$p_c^1\big|_{t=0} = \frac{(1-\phi)\delta}{(1+\phi)\delta - \phi} u \qquad (17)$$

Recall that in the homogeneous market all consumers have the same reservation price, u. Since $\delta < \delta_1(0,\phi) = \frac{1}{2}$, we have that $p_c^1|_{t=0} > u$ if $(1+\phi)\delta - \phi > 0$, and $p_c^1|_{t=0} < 0$ otherwise. Hence, the solution $p_c^1|_{t=0}$ is economically irrelevant and tacit collusion is impossible for all prices in $p \in [2t + \frac{t}{\phi}, u\ ]$ if $\delta < \frac{1}{2}$.

In the Appendix, we show that exactly the same happens in the differentiated market if t is not too large. We thus have

Proposition 2. *Suppose that goods are almost homogeneous. If the discount factor is less than the lowest discount factor allowing full collusion*

[8] The simulations are made in Mathematica and available on request from the author.

on the monopoly price, tacit collusion is impossible on all prices giving higher profit than the mixed strategy Nash equilibrium.

Again this result is the same as in a perfectly informed homogeneous market. The intuition is the same as in relation to Proposition 1, the relevant incentives only affect the informed market, and transparency only affects the size of the informed market. Concluding this section, transparency has no effect on the scope for collusion in a homogeneous market. Furthermore, in such a market collusion is either possible on the monopoly price or not at all. In a slightly differentiated market the latter is also true, and the effect of changes transparency on the scope for collusion is vanishingly small.

4.2 Relatively high product differentiation

We now consider the case where product differentiation is relatively high so that (5) is not fulfilled and the one shot Nash equilibrium is in pure strategies. First assume that (13) is not fulfilled. Inserting the relevant expressions, the non-deviation constraint for tacit collusion (12) becomes

$$\frac{1}{1-\delta}\frac{p^m}{2} \geq \frac{1}{8}\frac{\left(p^m + \frac{t}{\phi}\right)^2}{\frac{t}{\phi}} + \frac{\delta}{1-\delta}\frac{1}{2}\frac{t}{\phi}. \tag{18}$$

It is fulfilled when firms are sufficiently patient, namely when

$$\delta \geq \delta_2(t,\phi) \equiv \frac{p^m - \frac{t}{\phi}}{p^m + 3\frac{t}{\phi}} = \frac{\left(u - \frac{t}{2}\right) - \frac{t}{\phi}}{\left(u - \frac{t}{2}\right) + 3\frac{t}{\phi}}. \tag{19}$$

Clearly, $0 < \delta_2(t,\phi) < 1$ and $\delta_2(t,\phi)$ is increasing in the level of market transparency, ϕ. In this sense full collusion on the monopoly price is more difficult to sustain when the market is more transparent. We see that increasing market transparency affects collusion the same way as increasing the substitutability of products - for a given monopoly price. In the Hotelling model the substitutability also affects the monopoly price, and therefore the two effects do not coincide completely.

For $t/u = \frac{2\phi}{\phi+2}$, the right hand side equals zero, and full collusion can be sustained even with $\delta = 0$. In this case the one shot Nash equilibrium price equals the monopoly price.

Then consider the case where (13) is fulfilled. The optimal deviation price and profit are then given by the second expression in (10) and (11) respectively. Inserting in the no-deviation constraint (12) yields that the monopoly price can only be sustained if the discount factor at least equals

$$\delta_3(t,\phi) = \phi \frac{(2u-3t) - 2\frac{t}{\phi}}{(2u-3t)(1+\phi) - 2\frac{t}{\phi}}$$

In the Appendix we prove that $\frac{\partial \delta_3(t,\phi)}{\partial \phi} > 0$.

Suppose then that the discount factor is lower than the relevant crucial discount factor, δ_2 or δ_3. In this case, the most favorable equilibrium from the point of view of the firms involves a collusive price which exactly makes the non-deviation constraint fulfilled, i.e. the price solves

$$\frac{1}{1-\delta}\pi(p) = \pi^d(p) + \frac{\delta}{1-\delta}\pi^N \tag{20}$$

Assuming that the collusive price, p, is such that $p \leq 2t + \frac{t}{\phi}$, so that the first expression in (11) is relevant, we get two solutions by inserting the relevant expressions, the one shot Nash equilibrium price $p^N(\phi) = t/\phi$ and

$$p_c^2 = \frac{1+3\delta}{(1-\delta)}\frac{t}{\phi}, \tag{21}$$

which yields the highest profit possible given the constraint (20) should be fulfilled. Clearly, p_c^2 is decreasing in the market transparency and so is the profit (which equals $p_c^2/2$).

The price p_c^2 is derived under the assumption that $p \leq 2t + \frac{t}{\phi}$, it is therefore only valid if $\delta \leq \frac{\phi}{2+\phi}$.

Consider then the case where the relevant deviation profit is given by the second expression in (11). In this case, the maximal sustainable price is

$$p_c^3 = \frac{t}{\phi} + t\frac{1-\delta}{\left(1-\delta-\frac{\delta}{\phi}\right)} \tag{22}$$

Differentiating (22) it is readily seen that p is decreasing in ϕ. The formula is only valid for $p > 2t + \frac{t}{\phi}$, or $\delta \geq \frac{\phi}{2+\phi}$. In this section we have thus shown

Proposition 3. *Suppose goods are fairly differentiated, i.e. condition (5) is not fulfilled. The lowest discount factor compatible with full collusion on the monopoly price is increasing in the degree of market transparency. If the discount factor is so low that full collusion is impossible, then the highest price the firms are able to collude on is decreasing in the degree of market transparency.*

When goods are fairly differentiated increasing transparency makes collusion harder to sustain regardless of whether an optimal deviation captures only part of the whole market or the whole market. There is an important difference between the two cases though. In the first case, a change in transparency is equivalent to a reparametrization of product differentiation (apart from the effect on the monopoly price) as is clear from the expressions for δ_2 and p_2. This is *not* the case, when the optimal deviation captures the whole market, (cf δ_3 and p_3). In the first case, the optimal deviation price is determined by the elasticity of demand, cf equation (10), which depends on t/ϕ. In the second case, the optimal deviation price makes the demand of the other firm from well-informed consumers zero. This price only depends on the degree of substitutability of the products. Hence, while in some part of the parameter space, changing market transparency is like changing product differentiation, this is not true in general.

It is well known that collusion is easier to sustain if the punishment phase becomes harder. In working paper version of this paper optimal symmetric stick and carrot strategies a la Abreu (1986) are analyzed when products are relatively differentiated and optimal deviations do not capture the whole market. It is shown that results resemble those we obtain here. The smallest discount factor necessary for sustaining full collusion is increasing in transparency, for lower discount factors, the highest price sustainable is decreasing in transparency. Furthermore, changes in transparency are equivalent to changes in product differentiation. In a homogeneous market, the results the same as those above. The harshest symmetric punishment available is reversion to the mixed strategy Nash equilibrium.

5 Concluding remarks

This paper has analyzed how changes in market transparency on the consumer side of the market affect the scope for tacit collusion. There are two opposing effects involved: the temptation to undercut the other firm increases as the market becomes more transparent, but so does the toughness of the punishment. In the differentiated Hotelling market the first effect dominates, increasing transparency makes collusion more difficult and is pro-competitive. This is qualitatively different from the result obtained in the almost homogeneous market, where the effects on deviation profit and punishment profit almost balances each other, so that the effects of changes in transparency on tacit collusion are vanishingly small. These results suggest that the degree of differentiation of the market is important if one wishes to assess the competitive effects of transparency.

6 Appendix

Proof of $r(t) \to u$ **as** $t \to 0$ and $\lim_{t\to 0} \frac{\partial r}{\partial \phi} = 0$.

Suppose $r(t) < u - \frac{t}{2}$ and consider raising the price with $\varepsilon < u - \frac{t}{2} - r(t)$. The change in profit is

$$\begin{aligned} d\Pi &= \left(\phi\left(\frac{1}{2} - \frac{1}{2t}\int_{r+\varepsilon-t}^{r+\varepsilon} F_t(q)dq\right) + \frac{1-\phi}{2}\right)(r(t)+\varepsilon) \\ &\quad - \left(\phi\left(\frac{1}{2} - \frac{1}{2t}\int_{r-t}^{r} F_t(q)dq\right) + \frac{1-\phi}{2}\right) r(t) \end{aligned}$$

Consider this change as $t \to 0$. Using l'Hospital's rule we get

$$\lim_{t\to 0} d\Pi = \lim_{t\to 0}\left[\begin{array}{c} \left(\phi\left(\frac{1}{2}-\frac{F_t(r+\varepsilon-t)}{2}\right)+\frac{1-\phi}{2}\right)(r(t)+\varepsilon) \\ -\left(\phi\left(\frac{1}{2}-\frac{F_t(r-t)}{2}\right)+\frac{1-\phi}{2}\right)r(t) \end{array}\right] \tag{23}$$

$$= \frac{1-\phi}{2}\varepsilon > 0$$

Hence, we have that as $t \to 0$, $r(t) \to u$.

Since $r(t) \to u$ for all ϕ it also follows that $\lim_{t\to 0}\frac{\partial r}{\partial \phi} = 0.\square$

To prove that $\lim_{t\to 0}\frac{\partial E\pi_0}{\partial \phi} = -\frac{u}{2}$, we need some preliminary steps.

Proof of $\frac{\partial F_t}{\partial \phi}$ is bounded for all p, ϕ.

In the text it was shown that F_t fulfills (7), which is restated for convenience

$$E\pi_0 = \left(\phi\left(1-\frac{1}{2t}\int_{p-t}^{p+t}F_t(q)dq\right)+\frac{1-\phi}{2}\right)p. \tag{8}$$

We can rewrite this as

$$\frac{1}{2t}\int_{p-t}^{p+t}F_t(q)dq = 1-\frac{\frac{E\pi_0}{p}-\frac{1-\phi}{2}}{\phi} \tag{24}$$

Differentiating both sides wrt p and rearranging

$$\frac{\partial\left(\frac{1}{2t}\int_{p-t}^{p+t}F_t(q)dq\right)}{\partial p} = \frac{1}{2t}\left(F_t(p+t)-F_t(p-t)\right) = \frac{E\pi_0}{2\phi p^2} \tag{25}$$

Substituting into (24) then yields

$$\frac{1}{2t}\int_{p-t}^{p+t}F_t(q)dq = 1-\frac{\frac{p\phi}{t}\left(F_t(p+t)-F_t(p-t)\right)-\frac{1-\phi}{2}}{\phi}$$

which gives

$$\frac{1}{2t}\int_{p-t}^{p+t}F_t(q)dq + \frac{p}{t}\left(F_t(p+t)-F_t(p-t)\right) = 1+\frac{1-\phi}{2\phi}$$

Since this is fulfilled for all p in the support of F_t, it follows that $\frac{\partial F_t}{\partial \phi}$ is bounded. $\square$.

b. **Proof of** $\lim_{t\to 0}\frac{\partial r}{\partial \phi}=0$.

For clarity, we also explicitly write F_t as well as r as functions of ϕ in this proof.

By definition $F_t\left(r\left(t,\phi\right),\phi\right)=1$ for all ϕ. Hence

$$\frac{\partial F_t}{\partial r}\frac{\partial r}{\partial \phi}+\frac{\partial F_t}{\partial \phi}=0$$

so

$$\frac{\partial r}{\partial \phi}=\frac{-\frac{\partial F_t}{\partial \phi}}{\frac{\partial F_t}{\partial r}}$$

Recall $\lim_{t\to 0}r\left(t,\phi\right)=u$ for all ϕ. As $F_t\left(u,\phi\right)=1$ for all ϕ we have $\frac{\partial F_t(u,\phi)}{\partial \phi}=0$. As furthermore (by l'Hospital's rule)

$$\lim_{t\to 0}\frac{1}{2t}\int_{p-t}^{p+t}F_t\left(q\right)dq=F_0\left(p\right)$$

for all p in the support of F_t, (25) gives that $\lim_{t\to 0}\frac{\partial F_t}{\partial r}>0$. We therefore have that

$$\lim_{t=0}\frac{\partial r}{\partial \phi}=\lim_{t=0}\frac{-\frac{\partial F_t}{\partial \phi}}{\frac{\partial F_t}{\partial r}}=0$$

□

We are now in the position to prove

Proof of $\lim_{t\to 0}\frac{\partial E\pi_0}{\partial \phi}=-\frac{u}{2}$

Differentiating (8) yields

$$\begin{aligned}&\frac{\partial E\pi_0}{\partial \phi}\\ =\;&\left(\left(-\frac{1}{2t}\int_{r-t}^{r}F_t(q)dq\right)-\frac{\phi}{2t}\int_{r-t}^{r}\frac{\partial F_t(q)}{\partial \phi}dq-\frac{\phi}{2t}\left[F_t\left(r\right)-F_t\left(r-t\right)\right]\frac{\partial r}{\partial \phi}\right)r\\ &+\left(\phi\left(\frac{1}{2}-\frac{1}{2t}\int_{r-t}^{r}F_t(q)dq\right)+\frac{1-\phi}{2}\right)\frac{\partial r}{\partial \phi}\end{aligned}$$

Using l'Hospital's rule several times this reduces to

$$\frac{\partial E\pi_0}{\partial \phi}=\lim_{t\to 0}\left[\left(-\frac{1}{2}-\frac{\phi}{2}\frac{\partial F_t(r)}{\partial \phi}-\frac{\phi}{2}\frac{\partial F_t\left(r\right)}{\partial r}\frac{\partial r}{\partial \phi}\right)r+\left(\frac{1-\phi}{2}\right)\frac{\partial r}{\partial \phi}\right]$$

Recall that $\lim_{t\to 0} \frac{\partial F_t(r)}{\partial \phi} = 0$, $\lim_{t\to 0} r = u$, and $\lim_{t\to 0} \frac{\partial r}{\partial \phi} = 0$. We thus obtain

$$\lim_{t\to 0} \frac{\partial E\pi_0}{\partial \phi} = -\frac{u}{2}$$

□

Proof of: collusion is impossible if the discount factor is below δ_1 when t is small.

The price p_c^1 is negative, if the numerator and denominator of (16) have different signs. Consider first the case where $\delta < \frac{\phi}{1+\phi}$, then the denominator is negative. If

$$t < r(t) \frac{\delta}{(1-\delta)} \frac{1-\phi+2\phi\Delta(t)}{(1+\phi)} \tag{26}$$

then the numerator of (16) is positive, and p is then negative. Equation (26) is clearly fulfilled for small t.

For $\delta = \frac{\phi}{1+\phi}$, there are no solutions to (16) except in the degenerate case where the numerator is also zero, (where infinitely many solutions occur).

Consider then $\delta > \frac{\phi}{1+\phi}$, so that the denominator in (16) is positive. If (26) is not fulfilled, the numerator of (16) is negative and p_c^1 is negative. If (26) is fulfilled p_c^1 is positive. We now consider this case. Recall, that we are considering discount factors so that collusion on $u - \frac{t}{2}$ is impossible, and the expressions used in the derivation of p_c^1 are only valid if $p_c^1 < u - \frac{t}{2}$, i.e. if

$$\frac{\frac{\delta}{1-\delta} r(t) \left(\frac{1-\phi}{2} + \phi\Delta\right) - t\frac{1+\phi}{2}}{\frac{1}{2(1-\delta)} - \frac{1+\phi}{2}} < u - \frac{t}{2}$$

At δ^1, the left hand side equals the right hand side. Differentiating the left hand side wrt δ we find,

$$\frac{\partial lhs}{\partial \delta} = \frac{r(t)\phi^2 - 2r(t)\phi^2\Delta(t) - r(t)\phi + t + t\phi}{(\delta - \phi + \phi\delta)^2}$$

which is negative for

$$t < r(t)\phi \frac{1-\phi+2\phi\Delta(t)}{1+\phi} \tag{27}$$

For t fulfilling (27) therefore, the solution $p_c^1 > u - \frac{t}{2}$. The right hand side of (27) is increasing in Δ and $\lim_{t\to 0} r(t) = u$. Therefore (27) is fulfilled for sufficiently small t. □

Proof of $\frac{\partial \delta_3(t,\phi)}{\partial \phi} > 0$.

Differentiating one finds that the sign of $\frac{\partial \delta_3(t,\phi)}{\partial \phi}$ equals the sign of

$$(\phi u - 2t(1+\phi))\, 4u\phi + \left(3\phi^2 + 12\phi + 4\right) t^2 \tag{28}$$

This is a parable in t, which attains its minimum at $t = 4(1+\phi)\, u \frac{\phi}{3\phi^2+12\phi+4}$, where the value is $-4\phi^3 u^2 \frac{\phi-4}{3\phi^2+12\phi+4} > 0$, as $\phi < 4$. We conclude that $\frac{\partial \delta_3(t,\phi)}{\partial \phi} > 0$.□

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