

Dynamic R&D with Spillovers: Competition vs Cooperation¹

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Abstract

We investigate dynamic R&D for process innovation in a duopoly where firms may either undertake independent ventures or form a cartel for cost-reducing R&D investments. By comparing the profit and welfare performances of the two settings in steady state, we show that private and social incentives towards R&D cooperation coincide for all admissible levels of the technological spillovers characterising innovative activity.

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1 Introduction

The role of technological spillovers in shaping the incentives to conduct R&D for process innovation has attracted a wide attention in the existing literature on oligopoly theory. The most relevant contributions in this vein are those of d'Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Suzumura (1992) and Amir (2000), to mention only a few. A general appraisal of the advantages associated with R&D cooperation, and the related policy measures, can be found in Katz and Ordover (1990).¹

The theoretical debate on the private and social advantages generated by R&D cooperation was triggered by an analogous policy debate on the same issue, leading to the National Research Cooperation Act that passed in the US in 1984.² Then, following Katz (1986), a large body of literature has discussed the theoretical and empirical facets of welfare-improving technology policies based upon two forms of R&D cooperation, namely, R&D cartels and research joint ventures.³ Here, we shall briefly summarise the approaches adopted in d'Aspremont and Jacquemin (1988) and Kamien *et al.* (1992).

d'Aspremont and Jacquemin (1988) consider a homogeneous Cournot duopoly, where each firm enjoys a spillover from the rival in terms of the final outcome of R&D activity, in the following sense. To firm i , investing k_i costs an amount bk_i^2 , which captures the presence of decreasing returns to innovative activity, but the total effective R&D contributing to reduce firm i 's marginal cost c_i is in fact $K_i = k_i + \beta k_j$, where β is the technological externality generated from the rival's investment k_j . Therefore, given a generic initial marginal cost \bar{c} , we have $c_i = \bar{c} - K_i$. In Kamien *et al.* (1992), instead, the spillover effect is measured in terms of Dollars or Euro, in the sense that they assume each firm to have a concave R&D technology $f(Y_i)$, where $Y_i = y_i + \beta y_j$ is the effective R&D effort, comprehensive of the external effect, and the reduction in firm i 's marginal cost is given by $c_i = \bar{c} - f(Y_i)$. This technology is coupled with linear R&D costs equal to y_i for each firm.

¹The underlying relationship between innovation and market structure came to the fore even earlier, of course. To this regard, see Spence (1984) and Reinganum (1989), *inter alia*. The above mentioned contributions share with Brander and Spencer (1983) the concept of R&D as a cost-reducing activity, adding to the Brander-Spencer setup the possibility of information transmission or technological externalities.

²For the EU and Japan, see Goto and Wakasugi (1988) and the CE Commission (1990).

³A relatively scanty attention has been paid to the possibility that any form of R&D cooperation facilitates collusion, either in prices or in quantities. To this regard, see Martin (1995), Lambertini *et al.* (1998, 2002, 2003) and Cabral (2000).

In other terms, what changes from the first to the second model is the way chosen to make the setup concave. In the former case, concavity is achieved through a convex R&D cost function, while in the latter case the same property rests upon a concave R&D technology. Using $f(Y_i) = \sqrt{y_i + \beta y_j}$, Amir (2000) shows that the two models are isomorphic up to the transformation $k_i = y_i/\sqrt{b}$. For this reason, one can focus upon d'Aspremont and Jacquemin (1988). In particular, they compare two different games: one where firms behave noncooperatively in choosing both R&D efforts and output levels, the other where firms form a cartel in the R&D stage, choosing thus R&D investments so as to maximise joint profits in that stage only, while they continue to adopt a Nash behaviour in the market stage.⁴ Comparing the two setups, d'Aspremont and Jacquemin (1988) find that (i) for high spillover levels [$\beta > 1/2$], R&D investments - and also cost reduction, clearly - are higher under cooperative behaviour, and conversely for low spillovers; (ii) for high spillover levels [$\beta > 1/2$], social welfare is higher under cooperative behaviour, and conversely. Unfortunately, they also find that cartel profits are higher than noncooperative profits when spillovers are low [$\beta < 1/2$]. This yields an undesirable conflict between private and social incentives towards R&D cooperation (or cartelisation).⁵

The d'Aspremont-Jacquemin model, or some variation of it, has been used thereafter to investigate several related issues, e.g., the possibility of setting up research joint ventures in relation to absorptive capacity (Kamien and Zang, 2000), the efficiency comparison between Bertrand and Cournot behaviour with product differentiation (Qiu, 1997), the endogenisation of spillovers (Katsoulacos and Ulph, 1998; Poyago-Theotoky, 1999; Amir and Wooders, 1999, 2003) and the effects of increasing the number of firms in the market (Hinlopen, 2000).⁶

However, the above mentioned lack of overlapping between social and private incentives towards cooperation has remained unsolved. To tackle this problem, we adopt an explicitly dynamic approach to describe the R&D ac-

⁴They consider also a third case, in which firms cooperate both in the R&D stage and in the market stage; however, we are not interested in this case.

⁵The literature on this topic has also discussed the issue of equilibrium stability, as for low levels of R&D costs (i.e., low levels of parameter b) there exists no internal solution as second order conditions are not met. On this issue, see Henriques (1990), d'Aspremont and Jacquemin (1990), Qiu (1997) and Amir and Wooders (1998).

⁶A large amount of research has also been carried out on the empirical side. See Lambertini *et al.* (2004) and the references therein.

tivity aimed at process innovation, modelled as a differential game whose basic components are as close as possible to the original ones contained in d'Aspremont and Jacquemin (1988). As in their paper, we confine our attention to the alternative cases where firm either behave fully noncooperatively or build up a cartel in R&D investments. We compare steady state profits and social welfare at the subgame perfect equilibria of the two cases, finding that irrespective of the spillover level, R&D cooperation is preferable to noncooperative behaviour from both a private and a social point of view. Intuitively, this result stems from investment smoothing, which is carried out by firms over the time horizon of the dynamic setting, while it is utterly impossible to achieve in a static two-stage game where firms are compelled to invest one-shot the full amount of resources required to achieve the equilibrium efficiency level of their productive technology.

Moreover, we characterize the time path of relevant variables along the transition to the steady state, under the different regimes we investigate. In particular, we consider the linear approximation of the relevant dynamic systems, and we find its analytical solution. Then, we take into consideration the whole history of the dynamic economic system (i.e., from the initial state to the asymptotic allocation) and we evaluate flows in present value over the whole time span running from zero to the infinity. Doing so, we find that the coincidence (or the conflict) between individual and social incentive to R&D cartel depends on parameter configuration. In any case, the conflict is far from being the rule, as there exist admissible parameter regions wherein private and social incentives indeed coincide.

The remainder of the paper is structured as follows. Section 2 illustrates the dynamic setup. Independent ventures are investigated in section 3, while the performance of the R&D cartel is described in section 4. Section 5 compares the outcomes of the two different regimes, i.e., the independent venture and the R&D cartel, focusing on the steady state allocations. Section 5 briefly characterizes the total cartel allocation, that is the situation in which firms cooperate in both the R&D and in the market stage, and the allocation decided by a benevolent planner, aiming at maximising social welfare. While sections 3 to 6 focus on the steady state allocations, section 7 studies the time path of the variables along the transition to steady state. Section 7 contains some concluding remarks.

2 The setup

We consider a duopoly with homogeneous goods over continuous time, $t \in [0, \infty)$. In every instant, the market demand function writes as follows:

$$p(t) = A - q_1(t) - q_2(t). \quad (1)$$

Each firm i supplies the market through a technology characterised by a constant marginal cost. Accordingly, her instantaneous cost function is $C_i(c_i, q_i, t) = c_i(t)q_i(t)$. the marginal cost borne by firm i evolves over time as described by the following kinematic equation:

$$\frac{dc_i(t)}{dt} \equiv \dot{c}_i = c_i(t) [-k_i(t) - \beta k_j(t) + \delta], \quad (2)$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t , while parameter $\beta \in [0, 1]$ measures the positive technological spillover that firm i receives from the R&D activity of firm j . Parameter $\delta \in [0, 1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the ageing of technology. Equation (2) can be rewritten as follows:

$$\frac{\dot{c}_i}{c_i(t)} = -k_i(t) - \beta k_j(t) + \delta, \quad (3)$$

so as to highlight that the rate of change of firm i 's marginal cost over time is linear in the instantaneous investment efforts. That is, (2) is indeed adynamic version of the linear R&D technology employed by d'Aspremont and Jacquemin in the static model.

The instantaneous cost of setting up a single R&D laboratory is:

$$\Gamma(k, t) = b [k(t)]^2, \quad (4)$$

where $k(t)$ is the R&D effort carried out at time t within the laboratory, and b is a positive parameter. Now define the instantaneous R&D investment of firm i as $\Gamma_i(k_i, t)$. If firms undertake independent ventures (i.e., each firm sets up her own R&D division or laboratory), then:

$$\Gamma_i(k_i, t) = b [k_i(t)]^2. \quad (5)$$

In such a case, firms may behave either noncooperatively or collusively. Throughout the game, firms, consumers and the social planner discount future profits at the common and constant discount rate $\rho > 0$.

3 Independent ventures

In this setting, firms adopt a strictly noncooperative behaviour in choosing both the output levels and the R&D efforts, each firm operating her own R&D division. The Hamiltonian of firm i is:

$$\begin{aligned} \mathcal{H}_i(\mathbf{q}, \mathbf{k}, \mathbf{c}, t) = e^{-\rho t} \{ & [A - q_1(t) - q_2(t) - c_i(t)] q_i(t) - b [k_i(t)]^2 + \\ & - \lambda_{ii}(t) c_i(t) [k_i(t) + \beta k_j(t) - \delta] - \lambda_{ij}(t) c_j(t) [k_j(t) + \beta k_i(t) - \delta] \} \end{aligned} \quad (6)$$

where $\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c_j(t)$, and $\mathbf{q}, \mathbf{k}, \mathbf{c}$ are the vectors of control and state variables.

As a first step, we proceed to characterize its open-loop Nash equilibrium. As a second step, we will show that it is subgame perfect. As a third step, we will focus on the steady state allocation.

The relevant first order conditions (FOCs) for the optimum are:

$$\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial q_i(t)} = A - 2q_i(t) - q_j(t) - c_i(t) = 0 ; \quad (7)$$

$$\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial k_i(t)} = -2bk_i(t) - \lambda_{ii}(t)c_i(t) - \beta\lambda_{ij}(t)c_j(t) = 0 . \quad (8)$$

On the basis of the above FOCs on control variables, we can state the following result:

Lemma 1 *The open-loop Nash equilibrium of the game is subgame perfect.*

Proof. See Appendix 1. ■

The result of Lemma 1 derives from the fact that the game under consideration is a *perfect game*, in the sense of Leimann and Schmitendorf (1983), i.e., the closed-loop equilibrium collapses into the open-loop one. As a consequence, the open-loop equilibrium is strongly time consistent.⁷

The adjoint or co-state equations for the open-loop solution are:

$$-\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial c_i(t)} = \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho \lambda_{ii}(t) \Leftrightarrow \quad (9)$$

⁷The label ‘perfect game’ is due to Fershtman (1987), where one can find a general technique to identify any such games. Another class of games where open-loop equilibria are subgame perfect is investigated by Reinganum (1982). For further details, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7).

$$\begin{aligned}
\frac{\partial \lambda_{ii}(t)}{\partial t} &= q_i(t) + \lambda_{ii}(t) [k_i(t) + \beta k_j(t) + \rho - \delta] \\
-\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial c_j(t)} &= \frac{\partial \lambda_{ij}(t)}{\partial t} - \rho \lambda_{ij}(t) \Leftrightarrow \\
\frac{\partial \lambda_{ij}(t)}{\partial t} &= \lambda_{ij}(t) [k_j(t) + \beta k_i(t) + \rho - \delta]
\end{aligned} \tag{10}$$

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions

$$\lim_{t \rightarrow \infty} \lambda_{ij}(t) \cdot c_j(t) = 0, \quad i, j = 1, 2. \tag{11}$$

From FOCs (7-8) we have, respectively:

$$q_i^*(t) = \frac{A - q_j(t) - c_i(t)}{2}, \tag{12}$$

$$k_i^*(t) = -\frac{[\lambda_{ii}(t)c_i(t) + \beta \lambda_{ij}(t)c_j(t)]}{2b}. \tag{13}$$

While (12) has the usual appearance of a standard Cournot best reply function, the optimal R&D effort in (13) depends upon co-state variables. Such expression can be differentiated w.r.t. time to get the dynamic equation of $k_i(t)$:

$$\frac{dk_i(t)}{dt} \equiv \dot{k}_i = -\frac{c_i(t) \frac{\partial \lambda_{ii}(t)}{\partial t} + \lambda_{ii}(t) \frac{dc_i(t)}{dt} + \beta \left[c_j(t) \frac{\partial \lambda_{ij}(t)}{\partial t} + \lambda_{ij}(t) \frac{dc_j(t)}{dt} \right]}{2b} \tag{14}$$

with $\partial \lambda_{ii}(t)/\partial t$ and $\partial \lambda_{ij}(t)/\partial t$ obtaining from (9-10). Then, (14) can be further simplified by using

$$\lambda_{ii}(t) = -\frac{2bk_i(t) + \beta \lambda_{ij}(t) c_j(t)}{c_i(t)} \tag{15}$$

which obtains from (8).

As to the second co-state variable, its dynamic equation (10) must be treated autonomously⁸ and, by imposing stationarity, i.e., $\partial \lambda_{ij}(t)/\partial t = 0$, we

⁸That is, whenever the FOCs of firm i cannot determine the optimal value of the co-state variable attached to the rival's state dynamics, the co-state equation pertaining to the state variable of firm j has to be treated as an additional state equation, on which one must impose stationarity in equilibrium. For more details on this issue, see Başar and Olsder (1982, 1995²), Mehlmann (1988) and Dockner *et al.* (2000).

obtain $\lambda_{ij}(t) = 0$. This yields:

$$\dot{k}_i = -\frac{c_i(t)}{2b} \left[q_i(t) - \frac{2bk_i(t)}{c_i(t)} \right]. \quad (16)$$

Now we solve the system of best reply functions (12), yielding the Cournot-Nash output level of firm i as a function of state variables:

$$q_i(t) = \frac{A - 2c_i(t) + c_j(t)}{3} \quad (17)$$

which can be plugged into (16). After imposing the symmetry condition $c_j(t) = c_i(t) = c(t)$, we may characterise the dynamics of the R&D effort of firm i in terms of her own state and control variables only:

$$\dot{k}_i = \rho k_i(t) - \frac{c(t)[A - c(t)]}{6b}. \quad (18)$$

Let us move on to consider the steady state allocation. Imposing the stationarity condition $\dot{k}_i = 0$ we obtain:

$$k^{IV}(t) = \frac{c(t)[A - c(t)]}{6b\rho} \geq 0 \text{ for all } c(t) \in [0, A], \quad (19)$$

where the superscript IV stands for *independent ventures*.

Note that, in general, the level of $c(t)$ will depend upon the technological spillover β , so that we can write:

$$\frac{\partial k^{IV}}{\partial \beta} = \frac{[A - 2c(t)] \cdot \partial c(t) / \partial \beta}{6b\rho} \quad (20)$$

which, in principle, may take either sign, depending upon the relative size of A and $c(t)$ as well as the sign of $\partial c(t) / \partial \beta$. The steady state level of marginal cost $c(t)$ can be found by solving:

$$\dot{c} = -c(t) [k^{IV}(t)(1 + \beta) - \delta c(t)] = 0 \quad (21)$$

which yields:

$$c = 0; c = \frac{A(1 + \beta) \pm \sqrt{(1 + \beta)[A^2(1 + \beta) - 24b\delta\rho]}}{2(1 + \beta)} \quad (22)$$

All solutions in (22) are real if and only if $\delta\rho \leq A^2(1 + \beta) / (24b)$. If so, they also satisfy the requirement $c \in [0, A]$. We may prove the following:

Proposition 2 *Provided that $\delta\rho \leq A^2(1+\beta)/(24b)$, the steady state point*

$$\begin{aligned} c^{IV} &= \frac{A(1+\beta) - \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \\ k^{IV} &= \frac{\delta}{1+\beta} \end{aligned}$$

is the unique saddle point equilibrium of the game with independent ventures.

Proof. See Appendix 2. ■

Correspondingly, equilibrium output and profits are:

$$q^{IV} = \frac{A(1+\beta) + \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{6(1+\beta)}; \quad (23)$$

$$\pi^{IV} = \frac{A^2(1+\beta)^2 - 6b\delta[3\delta + 2\rho(1+\beta)] + A\sqrt{(1+\beta)^3[A^2(1+\beta) - 24b\delta\rho]}}{18(1+\beta)}, \quad (24)$$

From the steady state expressions of R&D investment, one can immediately derive the following intuitive property:

$$\frac{\partial k^{IV}}{\partial \beta} = -\frac{\delta}{(1+\beta)^2} < 0, \quad (25)$$

which implies that, as the size of technological spillover effects increases, the incentive to invest in process innovation shrinks as it becomes increasingly difficult to internalise the benefits from R&D activity. Observe that, in steady state, R&D investment is needed only to make up for the depreciation rate, with each firm receiving a positive externality from the other. Indeed, the total effective investment perceived from the viewpoint of firm i is $k^{IV}(1+\beta) = \delta$, which entails that the reduction of firm i 's individual R&D effort is fully made up for by the spillover effect. This, however, accounts for the specific functional form of k^{IV} in the steady state only, keeping in mind (20).

Moreover, in steady state the following also holds:

$$\frac{\partial c^{IV}}{\partial \beta} = -\frac{6b\delta\rho}{\sqrt{(1+\beta)^3[A^2(1+\beta) - 24b\delta\rho]}} < 0. \quad (26)$$

This is due to the fact that any increase in β entails a reduction in k^{IV} , as we know from (25). Indeed, using (20) and (25), we can write:

$$\frac{[A - 2c(t)] \cdot \partial c(t) / \partial \beta}{6b\rho} = -\frac{\delta}{(1 + \beta)^2} \quad (27)$$

which must hold in equilibrium. From the above condition, we obtain:

$$\frac{\partial c(t)}{\partial \beta} = -\frac{6b\delta\rho}{(1 + \beta)^2 [A - 2c(t)]}; \quad (28)$$

Then, noting that $A > 2c^{IV}$, it follows that $\partial c^{IV} / \partial \beta < 0$.

Consumer surplus and welfare in steady state are:

$$CS^{IV} \equiv \frac{(A - p^{IV}) \sum_{i=1}^2 q_i^{IV}}{2} = \frac{[A\sqrt{1 + \beta} + \sqrt{A^2(1 + \beta) - 24b\delta\rho}]^2}{18(1 + \beta)} \quad (29)$$

$$SW^{IV} \equiv 2\pi^{IV} + CS^{IV} = \frac{2 \left[A^2(1 + \beta)^2 - 3b\delta[3\delta + 4\rho(1 + \beta)] + A\sqrt{(1 + \beta)^3 [A^2(1 + \beta) - 24b\delta\rho]} \right]}{9(1 + \beta)}. \quad (30)$$

4 R&D cartel

Here, we examine the case where firms noncooperatively choose output levels, while maximising joint profits w.r.t. the choice of their respective R&D efforts. As in the previous section, each firm operates her own laboratory. This amounts to imposing *a priori* the symmetry conditions $k_i(t) = k_j(t) = k(t)$ and $c_i(t) = c_j(t) = c(t)$. The state equation now looks as follows:

$$\dot{c} = c(t) [-(1 + \beta)k(t) + \delta]. \quad (31)$$

Therefore, the Hamiltonian of firm i can be written as follows:

$$\begin{aligned} \mathcal{H}_i(\mathbf{q}, k, c, t) = e^{-\rho t} \{ & [A - q_1(t) - q_2(t) - c(t)] q_i(t) - b[k(t)]^2 + \\ & + \lambda(t)c(t) [-(1 + \beta)k(t) + \delta] \} \end{aligned} \quad (32)$$

where $\lambda(t) = \mu(t)e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c(t)$, and \mathbf{q} is the vector of individual outputs. The open-loop⁹ first order conditions for the optimum are:

$$\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial q_i(t)} = A - 2q_i(t) - q_j(t) - c(t) = 0 ; \quad (33)$$

$$\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial k(t)} = -2bk(t) - \lambda(t)(1 + \beta)c(t) = 0 ; \quad (34)$$

$$-\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial c(t)} = \frac{\partial \lambda(t)}{\partial t} - \rho\lambda , \quad (35)$$

along with the initial conditions $\{c(0)\} = \{c_0\}$, and the transversality conditions $\lim_{t \rightarrow \infty} \lambda(t) \cdot c(t) = 0$.

Solving (34), we obtain $\lambda(t) = -2bk(t)/[(1 + \beta)c(t)]$, entailing also:

$$\frac{dk(t)}{dt} \equiv \dot{k} = -\frac{(1 + \beta)}{2b} \left[c(t) \frac{\partial \lambda(t)}{\partial t} + \lambda(t) \frac{dc(t)}{dt} \right] . \quad (36)$$

From (35) we obtain:

$$\frac{\partial \lambda(t)}{\partial t} = q_i(t) - [\delta - \rho - (1 + \beta)k(t)]\lambda(t) . \quad (37)$$

This expression, together with the optimal values of the co-state variable and the Cournot-Nash output level $q^{CN} = [A - c(t)]/3$, can be plugged into (36), which simplifies as follows:

$$\dot{k} = \rho k(t) - \frac{c(t)[A - c(t)](1 + \beta)}{6b} . \quad (38)$$

Therefore, $dk(t)/dt = 0$ in correspondence of:

$$k^{Cl}(t) = \frac{c(t)[A - c(t)](1 + \beta)}{6b\rho} , \quad (39)$$

where superscript *Cl* stands for *cartel*. Before proceeding, we may compare $k^{Cl}(t)$ against $k^{IV}(t)$, as defined in (19), to ascertain that the following property:

$$\frac{c(t)[A - c(t)](1 + \beta)}{6b\rho} > \frac{c(t)[A - c(t)]}{6b\rho} \Rightarrow k^{Cl}(t) > k^{IV}(t) \quad (40)$$

holds for all $\beta \in (0, 1]$ and $c(t) > 0$. This entails:

⁹As in the previous case, it can be shown that the open-loop equilibrium is subgame perfect. The details are omitted for brevity, as they closely replicate the same line as in the proof of Lemma 1.

Lemma 3 *Setting up a cartel in the R&D stage leads firms to invest more than in the fully noncooperative case, for all admissible levels of spillovers and marginal cost.*

Of course, the reason for this result is that R&D cooperation permits to better internalise the beneficial externality, therefore boosting firms' incentives to invest. The consequence of the above Lemma is that the private and social desirability of R&D cooperation drastically hinges upon its ability of reducing marginal cost significantly below the level resulting from Nash behaviour.

Plugging $k^{Cl}(t)$ into the state dynamics and imposing the stationarity condition, we have:

$$\dot{c} = -c(t) \left[\frac{c(t) [A - c(t)] (1 + \beta)^2}{6b\rho} - \delta \right] = 0 \quad (41)$$

yielding:

$$c = 0; c = \frac{A(1 + \beta) \pm \sqrt{A^2(1 + \beta)^2 - 24b\delta\rho}}{2(1 + \beta)}. \quad (42)$$

The above analysis allows us to state:

Proposition 4 *Provided that $\delta\rho \leq A^2(1 + \beta)^2 / (24b)$, the steady state point*

$$c^{Cl} = \frac{A(1 + \beta) - \sqrt{A^2(1 + \beta)^2 - 24b\delta\rho}}{2(1 + \beta)}$$

$$k^{Cl} = \frac{\delta}{1 + \beta}$$

is the unique saddle point equilibrium of the game where firm set up a cartel in the R&D stage.

Proof. See Appendix 3. ■

The steady state R&D effort is exactly the same as in the noncooperative case. This is obviously due to the fact that, in both cases, the investment needed to keep constant firm i 's marginal cost is $k^m(1 + \beta) = \delta$, $m = IV, Cl$.

What changes, instead, is the steady state level of the marginal cost. To this regard, it can be easily verified that

$$c^{IV} - c^{Cl} \propto \frac{\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{1+\beta} - \frac{\sqrt{A^2(1+\beta) - 24b\delta\rho}}{\sqrt{1+\beta}} \quad (43)$$

which is strictly positive for all $\beta \in (0, 1]$.

Individual output and profits under R&D cartelisation are:

$$q^{Cl} = \frac{A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{6(1+\beta)}; \quad (44)$$

$$\pi^{Cl} = \frac{A^2(1+\beta)^2 - 6b\delta(3\delta + 2\rho) + A(1+\beta)\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{18(1+\beta)^2}. \quad (45)$$

Consumer surplus and welfare in steady state are:

$$CS^{Cl} \equiv \frac{(A - p^{Cl}) \sum_{i=1}^2 q_i^{Cl}}{2} = \frac{\left[A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho} \right]^2}{18(1+\beta)^2} \quad (46)$$

$$\begin{aligned} SW^{Cl} &\equiv 2\pi^{Cl} + CS^{Cl} = \\ &= \frac{2 \left[A^2(1+\beta)^2 - 3b\delta(3\delta + 4\rho) + A(1+\beta)\sqrt{A^2(1+\beta)^2 - 24b\delta\rho} \right]}{9(1+\beta)^2}. \end{aligned} \quad (47)$$

Propositions 2 and 4 immediately entail the following Corollary:

Corollary 5 *For all $\beta \in (0, 1]$ the parameter region wherein the R&D cartel problem admits an internal optimum is wider than the parameter region wherein noncooperative R&D activity yields an internal optimum.*

Proof. To show this, it suffices to verify that

$$A^2(1+\beta)^2 / (24b) > A^2(1+\beta) / (24b) \quad (48)$$

for all $\beta \in (0, 1]$. ■

This of course stems from the fact that R&D cooperation is substantially equivalent to a reduction in ρ (or δ , or both). That is, when investing within a cartel, firms behave as if they were more patient than in the alternative case.

5 Private and social incentives to R&D co-operation

Now we are in a position to assess the incentive to activate a cartel in the R&D stage, both from the standpoints of each firm and from the regulator's, in the parameter region where both organizational arrangements are admissible, i.e., $\delta\rho \leq A^2(1+\beta)/(24b)$. This task involves, respectively, evaluating π^{Cl} against π^{IV} and SW^{Cl} against SW^{IV} . In both cases, we obtain:

$$\pi^{Cl} - \pi^{IV} \propto SW^{Cl} - SW^{IV} \propto \Theta \quad (49)$$

$$\Theta \equiv 12b\beta\delta\rho + A(1+\beta) \left[\sqrt{A^2(1+\beta)^2 - 24b\delta\rho} - \sqrt{1+\beta} \sqrt{A^2(1+\beta) - 24b\delta\rho} \right]$$

with the expression Θ being positive for all $\beta \in (0, 1]$. Therefore, we have proved our main result:¹⁰

Proposition 6 *Consider the parameter range $\delta\rho \leq A^2(1+\beta)/(24b)$. For all positive spillover levels, the R&D cartel is preferable to independent ventures from private and social standpoints alike.*

As a final remark, we may observe that the beneficial effect of R&D cartelisation on social welfare comes from both sides of the market, since:

$$\pi^{Cl} > \pi^{IV} \text{ and } CS^{Cl} > CS^{IV} \quad (50)$$

for all $\beta \in (0, 1]$. This can be explained on the following grounds. Given that $c^{IV} - c^{Cl} > 0$, one expects firms to expand output under cooperative R&D, as against case where they undertake independent ventures. By comparing (23) and (44), there indeed emerges that $q^{Cl} > q^{IV}$ always. Accordingly, consumer surplus is enhanced by R&D cooperation because industry output is larger and market price is lower than in the fully noncooperative setting. As for the performance of firms, the increase in profits generated by the cartel arrangement is not obvious *a priori*, because the increase in productive efficiency is surely beneficial but the opposite holds for the output expansion. In balance, it appears that the first effects outweighs the second.

¹⁰Also note that, for all $\beta \in (0, 1]$, the parameter region where the cartel solution is admissible, i.e., $\delta\rho \leq A^2(1+\beta)^2/(24b)$, is wider than the analogous region defined for independent ventures to yield an admissible solution, i.e., $\delta\rho \leq A^2(1+\beta)/(24b)$.

The interpretation of these results (in particular, Proposition 6) is straightforward. With convex R&D costs, allowing firms to smooth their investment plans over some time span¹¹ is advantageous both because it permits firms to enhance profits and because it yields a higher welfare, as Jensen's inequality applied to the cost function $\Gamma_i(k_i, t)$ trivially implies. Moreover, the R&D effort in steady state is independent of the type of regimes, and it is dictated only by technological reason (the steady state effort level is such that it exactly neutralizes the depreciation of productive efficiency, and maintains a constant marginal cost of production).

6 The total cartel and the Social Planner cases

Just for the sake of a complete comparison with d'Aspremont and Jaquemin's paper, we solve the dynamic problem corresponding to the case in which firms cooperate both in the R&D stage, and in the market stage, aiming at the maximum present value of joint profit flows over time. We label this case as *total cartel (TC)*. The appropriate dynamic equation concerning the control variable k turns out to be:

$$\dot{k} = \rho k(t) - \frac{(1 + \beta)}{2b} cq(t) \quad (51)$$

and by substituting the relevant expression for q , that is, $q = (A - c)/4$ deriving from the appropriate FOC, we obtain:

$$\dot{k} = 0 \Rightarrow k^{TC} = \frac{c(A - c)(1 + \beta)}{8b\rho} \quad (52)$$

Similarly, we can consider the case of a benevolent *social planner (SP)* aiming at the maximum present value of social welfare over time. In this case, the appropriate dynamic equation concerning the control variable k remains the same as in (51), and by substituting the relevant expression for q , that is $q^* = (3A - 2c)/12$, we obtain:

$$\dot{k} = 0 \Rightarrow k^{SP} = \frac{c(3A - 2c)(1 + \beta)}{24b\rho} \quad (53)$$

¹¹The length of the horizon and the assumption that time be treated as a continuous variable are both, in fact, immaterial to the conclusions. It can be easily shown that an analogous differential game over $t \in [0, T]$ (no matter whether in continuous or discrete time) would produce qualitatively equivalent results.

A phase diagram in the (c, k) space can be easily provided, for all four considered settings: the *IV*, *Cl*, *TC*, and *SP* cases.

Irrespective of the regime, the $\dot{c} = 0$ locus corresponds to the horizontal line $k = \delta / (1 + \beta)$.

As far as the $\dot{k} = 0$ locus is concerned, in all the cases under consideration, it is a parabola crossing the $(0, 0)$ point, and increasing over the interval $c \in (0, A/2)$; moreover, it is immediate to note that, in the mentioned relevant interval:

- i] the $\dot{k}(IV) = 0$ locus lies below the $\dot{k}(Cl) = 0$ locus, so that the steady state level of c turns out to be lower under the *Cl* case, as compared to the *IV* case;
- ii] the $\dot{k}(TC) = 0$ locus lies below the $\dot{k}(Cl) = 0$ locus, while it can be above or below the $\dot{k}(IV) = 0$ locus, depending on whether β is larger or smaller than $1/3$, respectively. As a consequence, the steady state level of c is larger under the *TC* case as compared to the *Cl* case, while it can be smaller or larger than the steady state level of c associated to the *IV* case;
- iii] the $\dot{k}(SP) = 0$ locus lies below the $\dot{k}(Cl) = 0$ locus (provided that $c < A/2$), while it lies above the $\dot{k}(TC) = 0$ locus, so that the following sequence holds in steady state, across the three mentioned regimes: $c^{Cl} < c^{SP} < c^{TC}$.

An additional case that is not investigated in d'Aspremont and Jacquemin (1988) can be quickly dealt with, namely, the RJV (research joint venture) whereby firms evenly split the cost of a unique R&D lab and then compete in output levels. In such a case, the cost borne by a single firm is $\Gamma_i(k_i, t) = b[k_i(t)]^2 / 2$, and, intuitively, this solution is more convenient than the cartel, all else equal.

7 Evaluating the saddle paths to the steady state

So far, the analysis has focused upon the steady state allocations. It is possible, however, to extend the analysis in order to characterise the evolution

of the systems over $t \in [0, \infty)$. In order to obtain the complete analytical solution, we consider the linear approximation around the steady state of the dynamic systems, and we solve the differential equations so as to obtain the (saddle) time paths of the relevant variables over $t \in [0, \infty)$. Then, we compute the present value of investment flows over time, along with the present value of intertemporal firms' profits, consumer surplus and social welfare.

The above procedure leads us to obtain the results stated in the following:

Proposition 7 *Provided that the steady states exist, the saddle path dynamics of investment and cost over $t \in [0, \infty)$, under independent ventures regime and the R&D cartel, are, respectively:*

$$k^{(IV)}(t) = -\frac{c_0 - c^{IV}}{(1 + \beta)c_0} r^{IV} \cdot \exp(r^{IV}t) + k^{IV} \quad (54)$$

$$c^{(IV)}(t) = (c_0 - c^{IV}) \cdot \exp(r^{IV}t) + c^{IV} \quad (55)$$

$$k^{(Cl)}(t) = -\frac{c_0 - c^{Cl}}{(1 + \beta)c_0} r^{Cl} \cdot \exp(r^{Cl}t) + k^{Cl} \quad (56)$$

$$c^{(Cl)}(t) = (c_0 - c^{Cl}) \cdot \exp(r^{Cl}t) + c^{Cl} \quad (57)$$

Proof. See Appendix 4. ■

Considering (17) under symmetry conditions, it is immediate to obtain the time path of the individual level of production under both the regimes being considered, that is, $q^{(IV)}(t) = (A - c^{(IV)}(t))/3$, and $q^{(Cl)}(t) = (A - c^{(Cl)}(t))/3$. Moreover, by simple substitutions it is possible to compute the individual firm's profits, as well as consumer surplus and social welfare, at every instant $t \in [0, \infty)$, under independent ventures regime and the cartel. Then, it is possible to compute the appropriately discounted sum of these measures for t going from zero to infinity. The aggregate measures are outlined in the following Proposition:

Proposition 8 *Provided that (a) the steady state exists under both regimes, (b) steady states are approached along the saddle path, (c) all agents have the common discount rate ρ , then: (i) the sum of the present value of the*

investment flows of a firm over $t \in [0, \infty)$, under independent ventures and R&D cartel, are, respectively:

$$\begin{aligned}
K^{IV} &= \int e^{-\rho t} k^{(IV)}(t) dt = \\
&= \frac{k^{IV}}{\rho} + \left[\frac{-r^{IV}(c_0 - c^{IV})}{(1 + \beta)c_0} \right] / (\rho - r^{IV}) \\
K^{CL} &= \int e^{-\rho t} k^{(CL)}(t) dt = \\
&= \frac{k^{CL}}{\rho} + \left[\frac{-r^{CL}(c_0 - c^{CL})}{(1 + \beta)c_0} \right] / (\rho - r^{CL})
\end{aligned}$$

where r^{IV} , r^{CL} denote the negative eigenvalue associated to the appropriate dynamic system, i.e.:

$$\begin{aligned}
r^{IV} &= \frac{\rho - \sqrt{\rho^2 + 2(A - 2c^{IV})(1 + \beta)c^{IV}/(3b)}}{2} \\
r^{CL} &= \frac{\rho - \sqrt{\rho^2 + 2(A - 2c^{CL})(1 + \beta)^2c^{CL}/(3b)}}{2}
\end{aligned}$$

(ii) the present values of individual profit flows accruing to a single firm over

$t \in [0, \infty)$, under independent ventures and R&D cartel, are, respectively:

$$\begin{aligned}
\Pi^{IV} &= \int_0^{+\infty} e^{-\rho t} \pi^{(IV)}(t) dt = & (58) \\
&= \frac{A}{9\rho} \left[A + \frac{c^{IV}}{\rho} + \frac{c_0 - c^{IV}}{\rho - r^{IV}} \right] + \\
&\quad - \frac{2}{9} \left[\frac{(c_0 - c^{IV})^2}{\rho - 2r^{IV}} + \frac{(c^{IV})^2}{\rho} + \frac{2(c_0 - c^{IV})c^{IV}}{\rho - r^{IV}} \right] + \\
&\quad - b \left[\frac{(c_0 - c^{IV})^2 (r^{IV})^2}{(1 + \beta)^2 c_0^2 (\rho - 2r^{IV})} + \frac{(k^{IV})^2}{\rho} - \frac{2(c_0 - c^{IV})r^{IV} k^{IV}}{(1 + \beta)c_0(\rho - r^{IV})} \right]
\end{aligned}$$

$$\begin{aligned}
\Pi^{Cl} &= \int_0^{+\infty} e^{-\rho t} \pi^{(Cl)}(t) dt = & (59) \\
&= \frac{A}{9\rho} \left[A + \frac{c^{Cl}}{\rho} + \frac{c_0 - c^{Cl}}{\rho - r^{IV}} \right] + \\
&\quad - \frac{2}{9} \left[\frac{(c_0 - c^{Cl})^2}{\rho - 2r^{Cl}} + \frac{(c^{Cl})^2}{\rho} + \frac{2(c_0 - c^{Cl})c^{Cl}}{\rho - r^{Cl}} \right] + \\
&\quad - b \left[\frac{(c_0 - c^{Cl})^2 (r^{Cl})^2}{(1 + \beta)^2 c_0^2 (\rho - 2r^{Cl})} + \frac{(k^{Cl})^2}{\rho} - \frac{2(c_0 - c^{Cl})r^{Cl} k^{Cl}}{(1 + \beta)c_0(\rho - r^{Cl})} \right]
\end{aligned}$$

(iii) the present values of consumer surplus over $t \in [0, \infty)$, under independent ventures and R&D cartel, are, respectively:

$$\begin{aligned}
PVCS^{IV} &= \int_0^{+\infty} e^{-\rho t} CS^{(IV)}(t) dt = \\
&= \frac{2A^2}{9\rho} - \frac{4A}{9} \left[\frac{c^{IV}}{\rho} + \frac{c_0 - c^{IV}}{\rho - r^{IV}} \right] + \\
&\quad + \frac{2}{9} \left[\frac{(c_0 - c^{IV})^2}{\rho - 2r^{IV}} + \frac{(c^{IV})^2}{\rho} + \frac{2(c_0 - c^{IV})c^{IV}}{\rho - r^{IV}} \right] \\
PVCS^{Cl} &= \int_0^{+\infty} e^{-\rho t} CS^{(Cl)}(t) dt = \\
&= \frac{2A^2}{9\rho} - \frac{4A}{9} \left[\frac{c^{Cl}}{\rho} + \frac{c_0 - c^{Cl}}{\rho - r^{Cl}} \right] + \\
&\quad + \frac{2}{9} \left[\frac{(c_0 - c^{Cl})^2}{\rho - 2r^{Cl}} + \frac{(c^{Cl})^2}{\rho} + \frac{2(c_0 - c^{Cl})c^{Cl}}{\rho - r^{Cl}} \right]
\end{aligned}$$

(iv) the present values of social welfare over $t \in [0, \infty)$, under independent

ventures and R&D cartel, are, respectively:

$$\begin{aligned}
PVS^{IV} &= 2\Pi^{IV} + PVS^{IV} = & (60) \\
&= \frac{4A^2}{9\rho} - \frac{2A}{9} \left[\frac{c^{IV}}{\rho} + \frac{c_0 - c^{IV}}{\rho - r^{IV}} \right] + \\
&\quad - \frac{2}{9} \left[\frac{(c_0 - c^{IV})^2}{\rho - 2r^{IV}} + \frac{(c^{IV})^2}{\rho} + \frac{2(c_0 - c^{IV})c^{IV}}{\rho - r^{IV}} \right] + \\
&\quad - 2b \left[\frac{(c_0 - c^{IV})^2 (r^{IV})^2}{(1 + \beta)^2 c_0^2 (\rho - 2r^{IV})} + \frac{(k^{IV})^2}{\rho} - \frac{2(c_0 - c^{IV})r^{IV}k^{IV}}{(1 + \beta)c_0(\rho - r^{IV})} \right] \\
PVS^{Cl} &= 2\Pi^{Cl} + PVS^{Cl} = & (61) \\
&= \frac{4A^2}{9\rho} - \frac{2A}{9} \left[\frac{c^{Cl}}{\rho} + \frac{c_0 - c^{Cl}}{\rho - r^{Cl}} \right] + \\
&\quad - \frac{2}{9} \left[\frac{(c_0 - c^{Cl})^2}{\rho - 2r^{Cl}} + \frac{(c^{Cl})^2}{\rho} + \frac{2(c_0 - c^{Cl})c^{Cl}}{\rho - r^{Cl}} \right] + \\
&\quad - 2b \left[\frac{(c_0 - c^{Cl})^2 (r^{Cl})^2}{(1 + \beta)^2 c_0^2 (\rho - 2r^{Cl})} + \frac{(k^{Cl})^2}{\rho} - \frac{2(c_0 - c^{Cl})r^{Cl}k^{Cl}}{(1 + \beta)c_0(\rho - r^{Cl})} \right]
\end{aligned}$$

Clearly, a comparative assessment of the different present values across regimes can be carried out, and clearcut conclusions may obtain once the numerical values of parameters are known. More importantly, comparing (58) with (60), one can check that the conflict between individual and social incentives towards R&D cartelisation, noted by d'Aspremont and Jacquemin for the static game, does not necessarily hold in the present framework where we take into consideration the whole history of the industry, from any admissible vector of initial conditions to the (asymptotic) steady state allocation. For instance, under admissible parameter configurations, a gain in individual profits deriving from R&D cartel implies an increase in social welfare. This is shown in Appendix 5.

8 Concluding remarks

We have analysed dynamic R&D investments for cost-reducing innovation in a Cournot duopoly where firms may either compete or cooperate in the R&D phase. The foregoing analysis has shown that a unique stable (in the saddle sense) equilibrium exists in each setting.

By comparing the steady state profit and welfare performances of the industry in the two cases, there emerges that private and social incentives towards R&D cooperation coincide for all admissible levels of the technological spillovers characterising innovative activity, in the sense that cartelisation dominates competition from both standpoints over the whole admissible parameter range.

The setup employed in the present paper is a dynamic version of the static game examined in d'Aspremont and Jacquemin (1988). The drastic difference between our results and theirs relies upon smoothing the investment efforts over time. Moreover, the R&D effort consistent with steady state is independent of the market competition type, but it is dictated only by a technological reason. Both considerations are ruled out by definition in a static setting, where, consequently, the private incentive appears to be inconsistent with the social incentive. This is not the case for the steady state of a proper dynamic problem. Also the extension to account for the whole time history of the dynamic systems driving our model shows that the conflict between individual and social incentive is far from being the rule.

Appendices

Appendix 1: Proof of Lemma 1

Consider the closed-loop information structure. The relevant first order conditions (FOCs) remain (7) and (8).

Observe that (7) only contains firm i 's state variable, so that in choosing the optimal output at any time during the game firm i may disregard the current efficiency of the rival. That is, there is no feedback effect in the output choice. Conversely, at first sight there seem to be a feedback between the R&D decisions, as (8) indeed contains both state variables, at least for any positive spillover effect (Intuitively, if $\beta = 0$, then the two investment plans are completely independent and therefore it is apparent that no feedback effect operates.). The core of the proof consists in showing that no feedback effect are actually present, even for positive spillover levels.

Taking the above considerations into account, the adjoint or co-state equations, under the closed-loop information structure, are:

$$-\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial c_i(t)} - \frac{\partial \mathcal{H}_i(\cdot, t)}{\partial k_j(t)} \cdot \frac{\partial k_j^*(\cdot, t)}{\partial c_i(t)} = \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho \lambda_{ii}(t) \Leftrightarrow \quad (\text{a1})$$

$$\begin{aligned} \frac{\partial \lambda_{ii}(t)}{\partial t} &= q_i(t) + \lambda_{ii}(t) [k_i(t) + \beta k_j(t) + \rho - \delta] - \frac{\beta}{2b} \lambda_{ji}(t) [\lambda_{ij}(t) c_j(t) + \beta \lambda_{ii}(t) c_i(t)] \\ -\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial c_j(t)} - \frac{\partial \mathcal{H}_i(\cdot, t)}{\partial k_i(t)} \cdot \frac{\partial k_i^*(\cdot, t)}{\partial c_j(t)} &= \frac{\partial \lambda_{ij}(t)}{\partial t} - \rho \lambda_{ij}(t) \Leftrightarrow \quad (\text{a2}) \end{aligned}$$

$$\frac{\partial \lambda_{ij}(t)}{\partial t} = \lambda_{ij}(t) \left\{ [k_j(t) + \beta k_i(t) + \rho - \delta] - \frac{\beta}{2b} [2bk_i(t) + \lambda_{ii}(t) c_i(t) + \beta \lambda_{ij}(t) c_j(t)] \right\}$$

where

$$\frac{\partial \mathcal{H}_i(\cdot, t)}{\partial k_j(t)} \cdot \frac{\partial k_j^*(\cdot, t)}{\partial c_i(t)} \quad (\text{a3})$$

capture the feedback effects, and partial derivatives $\partial k_j^*(\cdot, t) / \partial c_i(t)$ are calculated using the optimal values of investments as from FOC (8):

$$k_j^*(\cdot, t) = -\frac{\lambda_{jj}(t) c_j(t) + \beta \lambda_{ji}(t) c_i(t)}{2b}. \quad (\text{a4})$$

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions (11).

From the adjoint equation (a2), we note that $\partial\lambda_{ij}(t)/\partial t = 0$ in $\lambda_{ij}(t) = 0$. Then, using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$k_i^*(., t) = -\frac{\lambda_{ii}(t)c_i(t)}{2b}, \quad (\text{a5})$$

which entails that $\partial k_i^*(., t)/\partial c_j(t) = 0$. By the underlying symmetry of the model, this holds for both firms, i.e., feedback (cross-)effects are nil along the equilibrium path. Thus, the adjoint equations pertaining to the closed-loop information structure turn out to coincide with the adjoint equation pertaining to the open-loop information, (9) and (10) respectively. Accordingly, the dynamic system under the closed-loop information structure coincides with the dynamic system under the open-loop information structure. As a consequence, the open-loop equilibrium is a degenerate closed-loop one, and it is strongly time consistent, or equivalently, subgame perfect. (It is also worth observing that this procedure shows that FOCs are indeed unaffected by initial conditions as well. The property whereby the FOCs on controls are independent of states and initial conditions after replacing the optimal values of the co-state variables is known as *state-redundancy*, and the game itself as *state-redundant* or *perfect*.)

Appendix 2: Proof of Proposition 2

Assume $\delta\rho \leq A^2(1+\beta)/(24b)$. The stability properties of the system (18-21) can be assessed by evaluating the trace and determinant of the following Jacobian matrix:

$$J_{IV} \equiv \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} = \delta - (1+\beta)k & \frac{\partial \dot{c}}{\partial k} = -(1+\beta)c \\ \frac{\partial \dot{k}}{\partial c} = -\frac{A-2c}{6b} & \frac{\partial \dot{k}}{\partial k} = \rho \end{bmatrix}$$

in correspondence of the steady state values of c and k . At $c = k = 0$, the trace is $T(J_{IV}) = \delta + \rho > 0$ and the determinant is $\Delta(J_{IV}) = \delta\rho > 0$. Therefore, such a point is unstable. In correspondence of

$$k = \frac{c[A-c]}{6b\rho}; c = \frac{A(1+\beta) + \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \quad (\text{a6})$$

we obtain $T(J_{IV}) = \rho > 0$ and

$$\Delta(J_{IV}) = \frac{A^2(1+\beta) - 24b\delta\rho + A\sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{12b} \quad (\text{a7})$$

which is clearly positive in the admissible parameter range. Finally, in

$$k = \frac{c[A-c]}{6b\rho}; \quad c = \frac{A(1+\beta) - \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \quad (\text{a8})$$

we have $T(J_{IV}) = \rho > 0$ again, and

$$\Delta(J_{IV}) = \frac{A^2(1+\beta) - 24b\delta\rho - A\sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{12b}. \quad (\text{a9})$$

In this case, $\Delta(J_{IV}) < 0$ in the admissible parameter region. Therefore, (a3) is the unique stable steady state point of the dynamic system; in particular, it is a saddle point. Simplifying the expression for optimal investment in steady state, we obtain $k^{IV} = \delta/(1+\beta)$.

Appendix 3: Proof of Proposition 4

Assume $\delta\rho \leq A^2(1+\beta)^2/(24b)$. The stability properties of the system (31-38) can be assessed by evaluating the trace and determinant of the following Jacobian matrix:

$$J_{Cl} \equiv \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} = \delta - (1+\beta)k & \frac{\partial \dot{c}}{\partial k} = -(1+\beta)c \\ \frac{\partial \dot{k}}{\partial c} = -\frac{(1+\beta)(A-2c)}{6b} & \frac{\partial \dot{k}}{\partial k} = \rho \end{bmatrix}$$

in correspondence of the steady state values of c and k . Observe that the only difference between J_{Cl} and J_{IV} is to be found in $\partial \dot{k}/\partial c$, since in the cooperative case this partial derivative fully embodies the spillover effect, which is absent in the previous case.

At $c = k = 0$, the trace is $T(J_{Cl}) = \delta + \rho > 0$ and the determinant is $\Delta(J_{Cl}) = \delta\rho > 0$. Therefore, such a point is unstable. In correspondence of

$$k = \frac{c[A-c]}{6b\rho}; \quad c = \frac{A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)} \quad (\text{a10})$$

we obtain $T(J_{Cl}) = \rho > 0$ and

$$\Delta(J_{Cl}) = \frac{\sqrt{A^2(1+\beta)^2 - 24b\delta\rho} \left[A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho} \right]}{12b} \quad (\text{a11})$$

which is clearly positive in the admissible parameter range. Finally, in

$$k = \frac{c[A-c]}{6b\rho}; \quad c = \frac{A(1+\beta) - \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)} \quad (\text{a12})$$

we have $T(J_{Cl}) = \rho > 0$ again, and

$$\Delta(J_{Cl}) = \frac{A^2(1+\beta)^2 - 24b\delta\rho - A(1+\beta)\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{12b}. \quad (\text{a13})$$

In this case, $\Delta(J_{Cl}) < 0$ in the admissible parameter region. Therefore, (a12) is the unique stable steady state point of the dynamic system; in particular, it is a saddle point. Simplifying the expression for optimal investment in steady state, we obtain again $k^{Cl} = \delta/(1+\beta) = k^{IV}$.

Appendix 4: Proof of Proposition 5

Under the case of independent ventures, the dynamic system describing the evolution of the industry under the open-loop Nash equilibrium is given by (2) and (18). This system produces a steady state, given by c^{IV}, k^{IV} as characterized by Proposition 2. The linear approximation of system (2-18) around its steady state is:

$$\dot{k}(t) = \rho k(t) - \frac{A - 2c^{IV}}{6b}c(t) + \frac{Ac^{IV} - 2(c^{IV})^2 - 6b\rho k^{IV}}{6b} \quad (\text{a14})$$

$$\dot{c}(t) = -(1+\beta)c^{IV}k(t) + c^{IV}[2(1+\beta)k^{IV} - \delta] \quad (\text{a15})$$

The associated eigenvalues are:

$$r_{1,2}^{IV} = \frac{\rho \pm \sqrt{\rho^2 + 2(A - 2c^{IV})(1+\beta)c^{IV}/(3b)}}{2}.$$

Clearly, one is positive while the other is negative, confirming that the steady state is a saddle. In order to select the stable arm, we consider only the negative eigenvalue, and denote it by r^{IV} for future reference. The time path of $k(t)$ and $c(t)$ has the following shape: $k(t) = me^{rt} + k^{IV}$, $c(t) = ne^{rt} + c^{IV}$, where constants m and n can be found using the initial conditions and the transversality conditions. Standard procedures lead to (54) and (55), as reported in Section 6.

Under the R&D cartel, the dynamic system describing the time evolution of the economy is given by (2) and (38). Its steady state is given by c^{Cl} , k^{Cl} as stated in Proposition 4. The linear approximation of this non-linear first order system around the steady state is

$$\dot{k}(t) = \rho k(t) - \frac{(1+\beta)(A-2c^{Cl})}{6b}c(t) + \frac{A(1+\beta)c^{Cl} - 2(1+\beta)(c^{IV})^2 - 6b\rho k^{Cl}}{6b} \quad (\text{a16})$$

$$\dot{c}(t) = -(1+\beta)c^{Cl}k(t) + c^{Cl}[2(1+\beta)k^{Cl} - \delta] \quad (\text{a17})$$

The eigenvalues are:

$$r_{1,2}^{Cl} = \frac{\rho \pm \sqrt{\rho^2 + 2(A-2c^{Cl})(1+\beta)^2c^{Cl}/(3b)}}{2}.$$

Again, one is positive and the other one is negative, as the steady state is a saddle. Since we take into account the stable arm, we consider only the negative value, denoted as r^{Cl} for future reference. The path of $k(t)$ and $c(t)$ over $t \in [0, \infty)$ is described by $k(t) = ve^{rt} + k^{Cl}$, $c(t) = ze^{rt} + c^{Cl}$, where constants v and z can be determined resorting to initial and transversality conditions. Standard procedures lead to (56) and (57), as in Section 6.

Appendix 5: Private vs social incentives to R&D cartelisation during the transition to the steady state

Consider first individual profits (in present value). The private incentive towards R&D cartelisation along the path to the steady state is measured by the difference between expression (59) and expression (58); define it as $\Delta\Pi^{Cl-IV}$. Likewise, the social incentive is measured by the difference between expression (61) and expression (60); define it as $\Delta PVSW^{Cl-IV}$. It is a matter

of tedious algebra to find:

$$\begin{aligned}\Delta\Pi^{Cl-IV} &= Z_1 + Z_2 + Z_3 \\ \Delta PVSW^{Cl-IV} &= -2Z_1 + Z_2 + 2Z_3\end{aligned}$$

where:¹²

$$\begin{aligned}Z_1 &= \frac{A}{9} \left[\frac{c^{Cl} - c^{IV}}{\rho} + \frac{c_0 - c^{Cl}}{\rho - r^{Cl}} - \frac{c_0 - c^{IV}}{\rho - r^{IV}} \right] \\ Z_2 &= -\frac{2}{9} \left[\frac{(c_0 - c^{Cl})^2}{\rho - 2r^{Cl}} - \frac{(c_0 - c^{IV})^2}{\rho - 2r^{IV}} + \frac{(c^{Cl})^2 - (c^{IV})^2}{\rho} + \right. \\ &\quad \left. + \frac{2(c_0 - c^{Cl})c^{Cl}}{\rho - r^{Cl}} - \frac{2(c_0 - c^{IV})c^{IV}}{\rho - r^{IV}} \right] \\ Z_3 &= -b \frac{1}{(1 + \beta)^2 c_0^2} \left(\frac{(c_0 - c^{Cl})^2 (r^{Cl})^2}{(\rho - 2r^{Cl})} - \frac{(c_0 - c^{IV})^2 (r^{IV})^2}{(\rho - 2r^{IV})} \right) + \\ &\quad + \frac{2k^{IV}}{(1 + \beta)c_0} \left(\frac{(c_0 - c^{Cl})r^{Cl}}{(\rho - r^{Cl})} - \frac{(c_0 - c^{IV})r^{IV}}{(\rho - r^{IV})} \right)\end{aligned}$$

Under the parameter configuration such that $Z_1 < 0$, one can check that $\Delta\Pi^{Cl-IV} > 0$ implies $\Delta PVSW^{Cl-IV} > 0$, while $\Delta PVSW^{Cl-IV} < 0$ implies $\Delta\Pi^{Cl-IV} < 0$. This is just an example of an admissible parameter subset for which there is no conflict between individual and social incentives to set up an R&D cartel, in the sense that if firms find the cartel to be convenient, the cartel turns out also to be convenient from a social welfare standpoint, while if the cartel is detrimental under a social welfare perspective, it turns out to be detrimental for the individual firm as well. Of course, under the assumption that $Z_1 < 0$, the case where the cartel is not convenient for firms while it is so from a social welfare point of view is admissible.

¹²Remember that $k^{IV} = k^{CL}$.

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