

Dynamic Price Discrimination With Asymmetric Firms*

Yongmin Chen

Department of Economics

University of Colorado

Boulder, CO 80309

March 28, 2006

Abstract. This paper provides an economic analysis of dynamic price discrimination in markets with asymmetric firms. Variants of a dynamic duopoly model are considered, where one firm has a stronger market position than its competitor. Firms observe consumers' past purchases, which may reveal consumers' different preferences towards the firms' products, due to brand differentiation and possibly also to the cost of switching suppliers. In equilibrium, the stronger firm has the incentive to offer prices based on consumers' purchase histories, which can intensify competition and reduce prices, at least for a short time. If price discrimination does not cause the weaker firm to exit, it tends to increase consumer welfare; otherwise it can reduce consumer welfare. The effect of price discrimination also depends on the differences in firms' (marginal) costs, the competitiveness of the market under uniform pricing, and consumers' time horizon. Without consumer switching cost, the stronger firm does not price below cost, whether its discriminatory price is pro- or anti-competitive. In the presence of switching cost, the stronger firm may price below cost in selling to the rival's customers, with the purpose and effect of eliminating competition.

*I thank Mark Armstrong, Mats Bergman, participants of the Seminar on Pros and Cons of Price Discrimination at the Swedish Competition Authority in November 2005, and participants of the ENCORE Workshop on Consumer Search and Switching Cost at the Erasmus University Rotterdam in February 2006 for helpful discussions and suggestions.

1. INTRODUCTION

In markets with repeated purchases, firms sometimes engage in price discrimination based on consumers' past purchases. For example, telephone companies sometimes offer lower prices to customers who switch from a competitor's service; credit card companies sometimes offer lower interest rates to consumers who transfer balances from competitors; and an electricity company may offer lower rates to a rival's customers. The discriminatory pricing in these examples has the common feature that the prices depend on consumers' past purchases and thus incorporate explicit dynamic considerations. Furthermore, the information about a consumer's purchase history takes a particularly simple form, namely whether the consumer has been purchasing from a competitor. Such dynamic pricing practices, sometimes also called behavior-based price discrimination, have received much attention in the recent economics literature (e.g., Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000; and Taylor, 2003). Unlike the traditional economic analysis about price discrimination, a common theme of this new literature is that dynamic price discrimination tends to intensify competition and benefit consumers.¹ This literature, however, has focused on markets where firms are ex ante symmetric and the number of firms is fixed exogenously.

In some recent antitrust cases, the issue of dynamic price discrimination has arisen in markets with asymmetric firms. For instance, in *AKZO*, the European Court of Justice upheld the principle established by an earlier decision of the European Commission that it is abusive for a dominant firm to offer selectively low prices to customers of a small competitor while maintaining substantially higher prices for its existing customers. The Court viewed such behavior as showing AKZO's adopting a strategy with the intention to damage its (small) competitor.² As another example, in 2005 the Swedish Competition Authority sued

¹Consumers can also benefit from price discrimination in static oligopoly (e.g., Holmes, 1989; and Corts, 1998). Fudenberg and Villas-Boas (2005) surveys behavior-based price discrimination. For a more general treatment of recent developments in the economics of price discrimination, see Armstrong (2005) and Stole (forthcoming).

²Judgment of the Court of Justice of 3 July 1991 in case C-62/86, *AKZO Chemie BV v Commission*, European Court Reports 1991, I-3359. See Chen (2005) for more discussion of this case, and of another

Teliasonera, a dominant firm in the Swedish telecom market, for having selectively offered better terms to a rival's customers. The agency alleges that the company has abused its dominant market position by engaging in the discriminating practices, in violation of the Swedish competition Act. These cases suggest a view by antitrust authorities that dynamic price discrimination in some asymmetric markets are anticompetitive and harmful to consumers, in contrast to the results from the existing economics literature. Therefore, it is desirable to extend the economics literature on dynamic price discrimination to markets with asymmetric firms, for the interests both in economic theory and in antitrust analysis.

This paper provides an economic analysis of dynamic price discrimination with asymmetric firms. We consider a market where there are two competing firms, one of which possesses a stronger brand (due to either higher quality in a vertical sense or a more desirable product position in a horizontal sense), which enables it to have a stronger (and possibly dominant) market position. Consumers have different brand preferences, and they may also have costs of switching suppliers.³ Time is discrete and there are $T \geq 3$ periods. The weaker firm may choose to exit the market after the two initial periods, due to a fixed cost to remain in the market; thus the number of active firms is endogenous. We study and compare the (subgame perfect) equilibrium of this market under uniform price (henceforth *UP*), where a firm can set only a single price to all consumers at each period, and the (perfect Bayesian) equilibrium of this market under discriminatory price (henceforth *DP*), where a firm can charge different prices to consumers who have or have not purchased from the rival. Our analysis offers several insights:

First, dynamic price discrimination tends to benefit consumers if it does not cause the weaker firm to exit. As in the existing literature, under *DP* competition is intensified for the more price-sensitive consumers after they are identified by past purchases. While this effect

relevant case, *Irish Sugar*.

³Dynamic price discrimination can occur both due to consumers' different brand preferences and due to consumer switching cost. In the former, Consumers' past purchases reveal information about their preferences towards different firms' products, enabling the firms to segment the consumers. In the latter, consumers are segmented from their past purchases even if they ex ante have no brand preferences. We consider both factors in this paper.

is likely to dominate on consumer welfare, there could be countervailing effects: the less price-sensitive consumers may pay more; and more subtly, attracted by the future benefits of low prices, some otherwise highly price sensitive consumers may initially become less sensitive to price differences between firms, resulting in a higher price on them early on. A long time horizon (i.e., both sufficiently high T and high discount factor δ) is sufficient for consumers to benefit from DP , or to reap the benefits of intensified competition under DP , if the number of firms does not decrease.

Second, dynamic price discrimination can harm consumers, if it causes the weaker firm to exit, after which the market is monopolized and, as a result, price increases. Since consumers may benefit from the lower price under DP before the weaker firm's exit, there could be a trade off between short-term gain and long-term loss in consumer welfare under DP . A sufficient condition for consumers to be worse off when DP causes exit is that consumers have a long time horizon, provided that the monopoly price by the stronger firm is higher than the equilibrium price under UP .

Third, the effects of dynamic price discrimination on consumers also depend importantly on the competitiveness of the market under uniform price and on the (marginal) cost difference between the two firms. If the degree of competition under UP is low, then DP is more likely to benefit consumers by intensifying competition. Furthermore, under DP the marginal cost of the stronger firm, holding the other firm's cost constant, affects consumer welfare in a non-monotonic way: Starting from the other firm's marginal cost level, increasing this cost initially reduces consumer welfare until the cost reaches some critical level, at which a further increase benefits consumers, and after that consumer welfare again decreases as this cost increases.

Fourth, in the absence of consumer switching cost, equilibrium price is weakly above average (variable) cost under price discrimination (which arises solely due to consumers' differences in brand preferences), even if discriminatory price causes exit of the weaker firm and harms competition. In the presence of switching cost, however, the stronger firm may offer below-cost prices to its weaker rival's customers, and such pricing can only serve the purpose of driving the rival from the market, with the effect of eliminating competition. This

suggests a sufficient, but not necessary, condition for identifying anticompetitive dynamic price discrimination by a dominant firm is that the firm engages in below-cost pricing in selling to its rival's customers.⁴ This underscores the importance of consumer switching cost in understanding firm behavior and market dynamics.⁵

Our paper is related to Armstrong and Vickers (1993), who consider a model where a dominant incumbent faces an endogenous degree of competition in one of its two markets. Price discrimination enables the incumbent to react more aggressively to entry, resulting in less entry, which tends to harm consumers, with the welfare effect depending on the relative efficiencies of the incumbent and the entrant. While our analysis shares some similar intuition as theirs, in our model consumers are strategic and their segmentation occurs endogenously, as a result of their past purchases. Thus, we incorporate dynamic considerations that are not present in their analysis.⁶ Our paper is also related to the economic theory and legal policy of predatory pricing, of which Bolton, Brodley, and Riordan (2000) provide a comprehensive treatment. For a price to be considered predatory, it is usually necessary that the price is below some measure of cost and the predator can later raise prices sufficiently to recoup the loss from below-cost pricing. Our analysis shows that when a dominant firm is able to identify a smaller rival's customers and to offer them a lower price, the price can be predatory without being below cost.

The rest of the paper is organized as follows. Section 2 describes and analyzes our basic model, which illustrates the trade off between *UP* and *DP* in a highly simplified

⁴We sometimes also call the firm with the stronger market position the "dominant" firm, when it has substantial market power.

⁵The competitive effects of switching costs have been studied extensively in the recent economics literature (e.g., Klemperer, 1987a; Farrell and Shapiro, 1988; and Farrell and Klemperer, forthcoming). Switching costs can alter the prices of an incumbent facing entry threat, and can explain limit pricing behavior, as in Klemperer (1987b). Our results show that switching costs can also alter the prices of a firm when there is the possibility of exit by its rival.

⁶For antitrust policy considerations, we focus on the effects of price discrimination on consumer welfare. While the possibility of dynamic price discrimination can also affect entry, our paper focuses on the effect of such practice on exit, motivated by antitrust cases where a dominant firm and its smaller competitor are both in the market in which dynamic price discrimination occurs.

framework. Section 3 extends the basic model to introduce increased competition under uniform price, which allows us to explore the effects of dynamic price discrimination more generally. Section 4 extends the basic model in another direction by allowing consumer switching cost, and identifies a key factor in determining whether and when anti-competitive below-cost pricing might arise in equilibrium. Section 5 concludes.

2. BASIC MODEL

Time is discrete and is indexed as $1, 2, \dots, T$, where $T \geq 3$. There are two firms, A and B , with constant marginal costs c_A and c_B , respectively, where $c_A \geq c_B$. For firm B , there is a fixed cost k to stay in the market for each of the periods $t = 3, \dots, T$.⁷ Other costs for A and B are normalized to zero.

There are two types of consumers, H and L . The total mass of consumers is equal to one, with the population size of types H and L being α and $1 - \alpha$, respectively. A consumer's type is her private information. Each type H consumer demands one unit of the product with reservation price V_H , and will only purchase from A . Each type L consumer will purchase $q = G(p)$ from the firm with the lower price, p , where

$$G(p) \begin{cases} > 0 & \text{if } p < V_H \\ = 0 & \text{if } p \geq V_H \end{cases} .$$

Each L -consumer receives surplus $s(p)$, and $s'(p) \equiv -G(p) < 0$ for $p < V_H$.⁸ The discount factor for all consumers and for both firms is δ .

This formulation captures in a highly simplified way a market where consumers have different brand preferences and one firm possesses a stronger brand that potentially enables

⁷Since our purpose is to model the potential exit decision of firm B , for ease of exposition we assume that there is no fixed cost for firm A . Also, we assume k occurs only after $t = 2$, so that our focus is on exit instead of entry decisions. We could alternatively assume that B needs to pay the fixed costs of the first two periods as an entry cost, and our results would be essentially the same as long as this entry cost is below B 's equilibrium profit for the first two periods.

⁸We could allow each H -consumer's demand to be downward-sloping as well, without substantially changing the analysis.

it to have a dominant position. One vertical-differentiation interpretation of the model is that A 's product has higher quality than B 's (consistent with $c_A \geq c_B$): H -consumers have sufficiently higher valuation for high quality than for low quality; but L -consumers do not place a premium on high quality, and their purchases are also more price sensitive (having a downward-sloping individual demand curve). We shall thus sometimes call H - and L -consumers the high- and low-value segments of the market, respectively. Alternatively, a horizontal-differentiation interpretation of the model is that the two firms are positioned at two separate locations with all consumers residing at A 's location (thus A is at an advantageous market position, but with higher marginal cost). H -consumers have sufficiently high transportation cost so that they only consider purchasing from A ; while L -consumers have zero transportation cost, and their demand is also more elastic. We shall call A the stronger firm and B the weaker firm. Both A and B will be strategic players of our model, and so will consumers.

Define, for $j = A, B$,

$$\pi_j(p) = (p - c_j)G(p), \quad (1)$$

$$p_j^m = \arg \max_p \pi_j(p). \quad (2)$$

Then p_j^m and $\pi_j(p_j^m)$ would be firm j 's monopoly price and profit with respect to L -consumers, if j were to be a monopolist selling to L -consumers. Notice that $p_j^m < V_H$.

Throughout the paper, we assume:

A1: For $j = A, B$, p_j^m exists uniquely, $c_A < p_B^m \leq p_A^m$, $\pi_j'(p) > 0$ for $p < p_j^m$, and $k \leq (1 - \alpha) \pi_B(p_B^m)$.

We note that *A1* contains familiar assumptions that are mostly self explaining. With $k \leq (1 - \alpha) \pi_B(p_B^m)$, we ensure that under uniform price firm B will not exit the market in equilibrium, which allows us to highlight how dynamic price discrimination may affect the exit decision of firm B .

We shall consider only pure strategies when equilibrium in pure strategies exists; we later expand the strategy set to include mixed strategies when there is no pure-strategy equilibrium.

For the basic model, we further assume that the following condition is satisfied:

$$(p_B^m - c_A) [\alpha + (1 - \alpha) G(p_B^m)] < \alpha (V_H - c_A). \quad (\text{C1})$$

The purpose of (C1) will become clear shortly.

We start our analysis by considering the equilibrium under uniform price, or *UP*. Each firm can only set a single price at each period under *UP*. Let $\{p_{tA}^u, p_{tB}^u\}$ be *A*'s and *B*'s prices at period t . The proper solution concept here is subgame perfect Nash equilibrium (SPNE).

At period T , the last period, the unique Nash equilibrium is $\{p_{TA}^{u*}, p_{TB}^{u*}\} = \{V_H, p_B^m\}$, with all *H* and *L* consumers purchasing from *A* and *B*, respectively. Clearly, *B* is maximizing its profit with p_B^m . Also, *A* cannot do better than charging V_H to sell only to the *H* consumers, because if it attempts to sell to the *L* consumers by undercutting *B*, its profit would be reduced due to condition (C1). The equilibrium profit for firm *B* in period T , without including k , is

$$(1 - \alpha) \pi_B(p_B^m).$$

By assumption *A1*, $(1 - \alpha) \pi_B(p_B^m) \geq k$, and hence it is optimal for *B* to stay in the market in period T . It follows that the unique subgame perfect equilibrium is for *B* to stay in the market all periods and for *A* and *B* to charge V_H and p_B^m at each period, or $\{p_{tA}^{u*}, p_{tB}^{u*}\} = \{V_H, p_B^m\}$ for $t = 1, \dots, T$.

Remark 1. Under *UP*, the basic model has a unique SPNE. At this equilibrium, *B* remains in the market, $\{p_{tA}^{u*}, p_{tB}^{u*}\} = \{V_H, p_B^m\}$, and *H*- and *L*-consumers purchase from *A* and *B* respectively for $t = 1, \dots, T$.

We next analyze equilibrium under discriminatory price, or *DP*. Under *DP*, firms can offer different prices to consumers who have different purchase histories.

Suppose that at the beginning of some $t \geq 2$, β_{tA} consumers have only purchased from *A* in the past, β_{tB} consumers have purchased from *B* in the past, and $1 - \beta_{tA} - \beta_{tB}$ have not made purchase in the past. Call them consumers with purchase history a , b , and o , respectively. Under *DP*, we assume that firm *A* and *B* each can offer up to three different

prices at every $t \geq 2$ to these three groups of consumers:

$$\left(p_{tA}^a, p_{tA}^b, p_{tA}^o\right) \text{ and } \left(p_{tB}^a, p_{tB}^b, p_{tB}^o\right).$$

A strategy of firm j under DP is a price at $t = 1$, p_{1j} , together with a sequence of prices $\left\{\left(p_{tj}^a, p_{tj}^b, p_{tj}^o\right) : t = 2, \dots, T\right\}$ if j is in the market, and for firm B there is also the decision of whether to exit at the end of $t \geq 2$.⁹

Firms have their beliefs about the probability that each consumer group belongs to type H at t , and we denote this probability that firm $j \in \{A, B\}$ assigns to group $i \in \{a, b, o\}$ by $\left(\mu_{tj}^a, \mu_{tj}^b, \mu_{tj}^o\right)$. Each consumer makes her purchase decision that maximizes her discounted sum of surplus.

A perfect Bayesian equilibrium (PBE), which is the proper solution concept for games under dynamic price discrimination, is a pair of two firms' strategies, a system of beliefs, and the purchase strategies of all consumers such that the strategies of all players are sequentially rational given the belief system, and the beliefs are consistent with players' strategies and with the Bayes' rule wherever possible. At any $t \geq 2$, if B believes that consumers with history a are all H customers, B cannot gain by offering any price to these consumers; thus, in this case, we will omit p_{tB}^a in describing B 's strategy. Also, if in equilibrium all consumers make purchases at any t , as they will in our model, p_{tj}^o will not be offered along the equilibrium path and is only relevant for out of equilibrium deviations. Unless otherwise stated, all equilibria in our analysis are supported by $\mu_{tj}^o = 1$ and $p_{tj}^o = V_H$ off the equilibrium path for $t \geq 2$; and to economize notations we shall also omit p_{tj}^o in describing equilibrium strategies.

As usual for games with price competition by firms having different costs, we shall assume that a firm will not bid any price for which its payoff is lowered if it succeeds in selling to the consumers at that price.

Proposition 1. (i) If $(1 - \alpha) \pi_B(c_A) \geq k$, the basic model has a unique PBE under DP .

⁹As it will become clear, once B exits, it has no incentive to re-enter the market, even if entry cost is zero. For ease of exposition, when we say B exits at the end of t , we also mean that B remains out of the market for the rest of the periods.

At this equilibrium, B stays in the market for all periods,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{tA}^{a*}, p_{tA}^{b*} \right), p_{tB}^{b*} \right\} &= \{(V_H, c_A), c_A\} \text{ for } t = 2, \dots, T; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in all periods.

(ii) If $(1 - \alpha) \pi_B(c_A) < k$, the basic model has a unique PBE under DP . At this equilibrium, B exits at the end of $t = 2$,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{2A}^{a*}, p_{2A}^{b*} \right), p_{2B}^{b*} \right\} &= \{(V_H, c_A), c_A\}, \\ \left(p_{tA}^{a*}, p_{tA}^{b*} \right) &= (V_H, p_A^m) \text{ for } t = 3, \dots, T; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in $t = 1, 2$, and all consumers purchase from A in $t = 3, \dots, T$.

Proof. (i) If in $t = 1$ both firms follow the equilibrium prices $\{p_{1A}^*, p_{1B}^*\}$ and consumers make the purchases as described, then A and B 's prices $\{(p_{tA}^{a*}, p_{tA}^{b*}), p_{tB}^{b*}\}$ at $t = 2, \dots, T$, and only these prices, are sequentially rational, following the equilibrium belief that consumers with histories a and b are H - and L -consumers, respectively, while the off-equilibrium belief and price are $\{\mu_{tj}^o = 1, p_{tj}^o = V_H\}$. It thus suffices to show that no player can profit from any deviation in $t = 1$, and that there can be no equilibrium where $\{p_{1A}^*, p_{1B}^*\} \neq \{V_H, p_B^m\}$. First, given the equilibrium prices and the behavior of other consumers, and given both firms' belief and prices off the equilibrium path, each consumer's purchase in $t = 1$ is optimal. Second, if A lowers its price sufficiently below p_B^m so that all L -consumers purchase from it in $t = 1$, at $t = 2$ A and B would face essentially the same game as in $t = 1$, but one period is lost and A 's profit in $t = 1$ is below the equilibrium profit, since by (C1)

$$(p_B^m - c_A) [\alpha + (1 - \alpha) G(p_B^m)] < \alpha (V_H - c_A).$$

Thus A cannot profit from any deviation that lowers its price at $t = 1$, and obviously it cannot benefit from raising its price above V_H . Third, B cannot benefit from deviating its

price away from p_B^m at $t = 1$. Finally, there can be no other equilibrium, since if there were another equilibrium, $\{\tilde{p}_{1A}^*, \tilde{p}_{1B}^*\}$, it must be that $\{\tilde{p}_{1A}^*, \tilde{p}_{1B}^*\} \neq \{V_H, p_B^m\}$. But then either A or B or both firms could increase payoff by deviating to $\{V_H, p_B^m\}$.

(ii) For any $t = 2, \dots, T$, if all H and L consumers purchase from A and B respectively in $t = 1$, B remains in the market in t , the unique equilibrium outcome in t is $\{(p_{2A}^{a*}, p_{2A}^{b*}), p_{tB}^{b*}\} = \{(V_H, c_A), c_A\}$; and it follows that, since $(1 - \alpha)\pi_B(c_A) < k$, B will stay out of the market for $t = 3, \dots, T$. Thus, it suffices to show that there is a unique equilibrium in which, with the equilibrium outcome in the continuation game, we have that $\{p_{1A}^*, p_{1B}^*\} = \{V_H, p_B^m\}$, and all H and L consumers purchase from A and B in $t = 1$, respectively. With the similar reasoning as in (i), neither any consumer, nor firm A or firm B , can benefit from any deviation in $t = 1$, and there can be no equilibrium with $\{p_{1A}^*, p_{1B}^*\} \neq \{V_H, p_B^m\}$. *Q.E.D.*

We note that our assumption that H -consumers are loyal to A and will only purchase from A significantly simplifies the analysis, but it is not essential for our results. If H -consumers could also consider purchasing from B , our results would be essentially the same if, for instance, (i) each H -consumer also has a downward-sloping demand curve and V_H is the monopoly price, and (ii) an H -consumer has higher surplus from A 's product under V_H than from B 's product under p_B^m due to vertical product differentiation.

Denote the discounted sum of aggregate consumer surplus under UP and DP by W^u and W^d , respectively. We have:

Corollary 1. In the basic model, $W^d > W^u$ if $(1 - \alpha)\pi_B(c_A) \geq k$ or if $c_A = c_B$; and $W^d < W^u$ if $(1 - \alpha)\pi_B(c_A) < k$, $c_A > c_B$, and both T and δ are sufficiently large. Furthermore, firm B is always worse off under DP .

Proof. We first notice that the H -consumers receive the same surplus (equal to zero) under both UP and DP . Thus, the change in aggregate consumer surplus is the same as the change in consumer surplus by the L -consumers.

Next, if $(1 - \alpha) \pi_B(c_A) \geq k$, since $c_A < p_B^m$,

$$\begin{aligned} W^d - W^u &= (1 - \alpha) \{ [s(p_B^m) + s(c_A)(\delta + \dots + \delta^{T-1})] - [s(p_B^m)(1 + \delta + \dots + \delta^{T-1})] \} \\ &= (1 - \alpha) [s(c_A) - s(p_B^m)] (\delta + \dots + \delta^{T-1}) > 0 \text{ for any } c_A \geq c_B. \end{aligned}$$

Next, if $(1 - \alpha) \pi_B(c_A) < k$,

$$\begin{aligned} W^d - W^u &= (1 - \alpha) [s(p_B^m) + \delta s(c_A) + s(p_A^m)(\delta^2 + \dots + \delta^{T-1})] - (1 - \alpha) [s(p_B^m)(1 + \delta + \dots + \delta^{T-1})] \\ &= (1 - \alpha) \delta [s(c_A) - s(p_B^m)] - [s(p_B^m) - s(p_A^m)] (\delta^2 + \dots + \delta^{T-1}). \end{aligned}$$

Thus, if $c_A = c_B$, $W^d - W^u = (1 - \alpha) \delta [s(c_A) - s(p_B^m)] > 0$, and if $c_A > c_B$

$$W^d - W^u \rightarrow (1 - \alpha) \delta \left\{ [s(c_A) - s(p_B^m)] - [s(p_B^m) - s(p_A^m)] \frac{\delta}{1 - \delta} \right\} < 0$$

for $T \rightarrow \infty$ and $\delta \rightarrow 1$.

Finally, it is straightforward that firm B' 's discounted sum of profits is lower under DP than under UP . *Q.E.D.*

The results here capture the basic (potential) conflict for consumer welfare under dynamic price discrimination with asymmetric firms. On the one hand, DP intensifies competition (for L -consumers), which by itself benefits consumers. This is an insight from the existing studies of dynamic price discrimination, where the focus has been on markets with ex ante symmetric firms. On the other hand, the intensified competition under DP can cause the weaker firm to exit the market, resulting in a price increase afterwards and potentially hurting consumers. In the simple model here, consumer welfare is higher under DP if it does not cause firm B to exit or if A' 's marginal cost is not higher than B' 's; while consumer welfare is lower under DP only if it causes firm B to exit, A has higher marginal cost, and the time horizon is sufficiently long (both T and δ are large).

Interestingly, because of this conflict, consumer welfare under DP , W^d , is non-monotonic in c_A . When c_A starts from some high value such that $(1 - \alpha) \pi_B(c_A) > k$, marginal decreases in c_A increase W^d . As c_A goes down further to reach the point at which $(1 - \alpha) \pi_B(c_A) =$

k , a marginal decrease in c_A reduces W^d , due to the exit effect. W^d rises again as c_A decreases further beyond that point.

We note that firm A 's lowest price in equilibrium is c_A , which is A 's marginal as well as average (variable) cost.

Remark 2. In equilibrium, firm A does not engage in below-cost pricing, even when A 's more aggressive pricing under DP induces firm B to exit and harms consumers.

Our basic model has two noteworthy features. First, due to condition (C1), competition is weak, in the sense that under uniform price the firm with a stronger brand relinquishes the low-value consumer segment to the weaker competitor, so that there is no head-to-head competition between the two firms. Price discrimination eliminates this market segmentation and intensifies competition. This formulation makes the best case for DP in its effect on consumer welfare. Second, consumer preferences are constant overtime; namely past purchases do not affect consumer preferences towards either firm. The basic model provides a benchmark for our analyses in the next two sections that further consider the effects of increased competition and of consumer switching cost (under which a consumer's preference towards different firms is affected by her past purchase).

3. INCREASED COMPETITION

We consider the effects of increased competition by introducing effective competition to the model under UP . The main difference from the basic model is that condition (C1) does not hold. That is, in this section we instead assume

$$(p_B^m - c_A) [\alpha + (1 - \alpha) G(p_B^m)] > \alpha (V_H - c_A). \quad (\text{C2})$$

In this case, let $\underline{p} \in (c_A, p_B^m]$ be such that

$$(\underline{p} - c_A) (\alpha + (1 - \alpha) G(\underline{p})) = \alpha (V_H - c_A). \quad (3)$$

Notice that the unique existence of \underline{p} is guaranteed since $\pi'_j(p) > 0$ for $p < p_j^m$. For this section, we further assume:

A2. (i) $s(\underline{p}) - s(p_B^m) < \delta s(c_A)$, and (ii) $k \leq (1 - \alpha) \pi_B(\underline{p})$.

Part (i) of A2 is easily satisfied if δ is not too small, which is a natural assumption since we are mainly interested in situations where the future is relatively important. Part (ii) strengthens part of Assumption A1, again ensuring that firm B will not exit in equilibrium under UP .¹⁰

Everything else is the same as in the basic model. Again, we first consider uniform price and then discriminatory price.

3.1 Uniform Price

From familiar arguments, there will be no pure-strategy Nash equilibrium for the stage game at each period. We thus consider mixed strategies. Consider the following pair of mixed strategies. Firm A chooses V_H and $p \in [\underline{p}, p_B^m]$ according to $F_A(p)$; and Firm B chooses $p \in [\underline{p}, p_B^m]$ according to $F_B(p)$. Firm A 's expected profit from choosing any $p \in [\underline{p}, p_B^m]$ is

$$\alpha(p - c_A) F_B(p) + (p - c_A) [\alpha + (1 - \alpha) G(p)] [1 - F_B(p)] = \alpha(V_H - c_A).$$

That is,

$$(p - c_A) (1 - \alpha) G(p) F_B(p) = (p - c_A) [\alpha + (1 - \alpha) G(p)] - \alpha(V_H - c_A),$$

and thus

$$F_B(p) = \frac{(p - c_A) (1 - \alpha) G(p) - \alpha(V_H - p)}{(p - c_A) (1 - \alpha) G(p)} \text{ for } \underline{p} < p < p_B^m.$$

Firm B 's expected profit from choosing any $p \in [\underline{p}, p_B^m]$ is

$$0 \cdot F_A(p) + (1 - \alpha) \pi_B(p) [1 - F_A(p)] = (1 - \alpha) \pi_B(\underline{p}).$$

Thus

$$F_A(p) = \frac{\pi_B(p) - \pi_B(\underline{p})}{\pi_B(p)} = 1 - \frac{\pi_B(\underline{p})}{\pi_B(p)} \text{ for } \underline{p} \leq p \leq p_B^m.$$

¹⁰If $c_A = c_B$, as $k \rightarrow 0$ and $\alpha \rightarrow 0$, the model has its limit the symmetric Bertrand duopoly under market demand $G(p)$.

Define

$$\begin{aligned}
F_A(p) &= \begin{cases} 1 & \text{if } p \geq V_H \\ 1 - \frac{\pi_B(\underline{p})}{\pi_B(p_B^m)} & \text{if } p_B^m \leq p < V_H \\ 1 - \frac{\pi_B(\underline{p})}{\pi_B(\underline{p})} & \text{if } \underline{p} \leq p < p_B^m \\ 0 & \text{if } p < \underline{p} \end{cases}, \\
F_B(p) &= \begin{cases} 1 & \text{if } p \geq p_B^m \\ 1 - \frac{\alpha(V_H - p)}{(1 - \alpha)\pi_A(p)} & \text{if } \underline{p} < p < p_B^m \\ 0 & \text{if } p < \underline{p} \end{cases}.
\end{aligned} \tag{4}$$

Proposition 2. Assume (C2) holds. Then there is a unique SPNE in mixed strategies under *UP*, at which firm *B* stays in the market all periods, with firms *A* and *B* pricing according to probability distribution functions $(F_A(p), F_B(p))$ in each period. The equilibrium profits per period for *A* and *B* are

$$\begin{aligned}
\pi_A^* &= \alpha(V_H - c_A), \\
\pi_B^* &= (1 - \alpha)\pi_B(\underline{p}).
\end{aligned}$$

Proof. First consider period *T*. For firm *A*, with $p = V_H$ or with any $p \in [\underline{p}, p_B^m]$, *A*'s expected profit given $F_B(p)$ is

$$\begin{aligned}
&\alpha(p - c_A)F_B(p) + (p - c_A)[\alpha + (1 - \alpha)G(p)][1 - F_B(p)] \\
&= (p - c_A)[\alpha + (1 - \alpha)G(p)] - (1 - \alpha)(p - c_A)G(p)F_B(p) \\
&= (p - c_A)[\alpha + (1 - \alpha)G(p)] - (1 - \alpha)\pi_A(p) \left[1 - \frac{\alpha(V_H - p)}{(1 - \alpha)\pi_A(p)} \right] \\
&= (p - c_A)\alpha + (1 - \alpha)\pi_A(p) - (1 - \alpha)\pi_A(p) + \alpha(V_H - p) = \alpha(V_H - c_A),
\end{aligned}$$

and *A*'s profit is lower with any other price. Thus, given $F_B(p)$, it is optimal for *A* to randomize on $p = V_H$ and on $p \in [\underline{p}, p_B^m]$ according to $F_A(p)$, with zero probability density placed on any other price.

For firm *B*, with any $p \in [\underline{p}, p_B^m]$, *B*'s expected profit given $F_A(p)$ is

$$\begin{aligned}
&0 + (1 - \alpha)(p - c_B)G(p)[1 - F_A(p)] = (1 - \alpha)\pi_B(p)[1 - F_A(p)] \\
&= (1 - \alpha)\pi_B(p) \frac{\pi_B(\underline{p})}{\pi_B(p)} = (1 - \alpha)\pi_B(\underline{p}),
\end{aligned}$$

and B 's profit is lower with any other price. Thus, given $F_A(p)$, it is optimal for B to randomize on $p \in [\underline{p}, p_B^m]$ according to $F_B(p)$, with zero probability density placed on any other price.

Notice that $F_A(\underline{p}) = 0$, $F_A(V_H) = 1$, and $F_A(p)$ has only one mass point at V_H with probability measure $\frac{\pi_B(\underline{p})}{\pi_B(p_B^m)}$. Notice also that $F_B(\underline{p}) = 0$, $F_B(p_B^m) = 1$, and $F_B(p)$ has only one mass point at p_B^m with probability measure $\frac{\alpha(V_H - p_B^m)}{(1-\alpha)\pi_A(p_B^m)} \in (0, 1)$. Hence $F_A(p)$ and $F_B(p)$ are genuine cumulative distribution functions. Thus, if B stays in the market, $(F_A(\cdot), F_B(\cdot))$ constitutes a mixed-strategy Nash equilibrium in period T . The equilibrium profits for A and B in $t = T$ are $\pi_A^* = \alpha(V_H - c_A)$ and $\pi_B^* = (1 - \alpha)\pi_B(\underline{p})$. Furthermore, there can be no other equilibrium. This is because if there were another equilibrium, $(\tilde{F}_A(\cdot), \tilde{F}_B(\cdot))$, the equilibrium payoff for A and B must still be $\alpha(V_H - c_A)$ and $(1 - \alpha)\pi_B(\underline{p})$, respectively. Writing down the expressions for expected profits of A and B , we would have $(\tilde{F}_A(\cdot), \tilde{F}_B(\cdot)) = (F_A(\cdot), F_B(\cdot))$. Thus the equilibrium in T is unique.

Since by assumption $A2$, $k \leq (1 - \alpha)\pi_B(\underline{p})$, B will indeed stay in the market in period T . Following backward induction, there is a unique subgame perfect Nash equilibrium in mixed strategies for the game, in which B stays in the market for all periods, A and B price according to $(F_A(\cdot), F_B(\cdot))$ in each period, and their equilibrium profits at each period are π_A^* and π_B^* . *Q.E.D.*

We note that the expected equilibrium prices of A and B at each period are:

$$p_A^u \equiv \int p dF_A(p) < V_H, \quad p_B^u \equiv \int p dF_B(p) < p_B^m.$$

Since H -consumers will only purchase from A while L -consumers will purchase from the firm with the lower price, and since there is positive probability $p_A < p_B$, at each period, the expected price of H -consumers is $p_H^u = p_A^u$, and the expected price of L -consumers is $p_L^u = p^u < p_B^u$.

Let the probability distribution of the lower price of the two firms at each period be

$F_{\min}(p)$. The expected discounted sum of aggregate consumer surplus is

$$\begin{aligned} W^u &= \left[\alpha (V_H - p_A^u) + (1 - \alpha) \int s(p) dF_{\min}(p) \right] (1 + \delta + \dots + \delta^{T-1}) \\ &> \left[\alpha (V_H - p_A^u) + (1 - \alpha) \int s(p) dF_B(p) \right] (1 + \delta + \dots + \delta^{T-1}) \\ &\geq [\alpha (V_H - p_A^u) + (1 - \alpha) s(p_B^u)] (1 + \delta + \dots + \delta^{T-1}), \end{aligned}$$

where the first inequality is due to $\int s(p) dF_B(p) < \int s(p) dF_{\min}(p)$, and the second inequality is because $s(p)$ is convex and thus

$$\int s(p) dF_B(p) \geq s\left(\int p dF_B(p)\right) = s(p_B^u).$$

On the other hand, we have

$$\begin{aligned} W^u &= \left[\alpha (V_H - p_A^u) + (1 - \alpha) \int s(p) dF_{\min}(p) \right] (1 + \delta + \dots + \delta^{T-1}) \\ &< [\alpha (V_H - p_A^u) + (1 - \alpha) s(\underline{p})] (1 + \delta + \dots + \delta^{T-1}). \end{aligned}$$

3.2 Effects of Price Discrimination

We first characterize equilibrium under price discrimination.

Proposition 3. (i) If $(1 - \alpha) \pi_B(c_A) \geq k$, there is a PBE under DP , at which B stays in the market for all periods,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{tA}^{a*}, p_{tA}^{b*} \right), p_{tB}^{b*} \right\} &= \{(V_H, c_A), c_A\} \text{ for } t = 2, \dots, T; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in all periods.

(ii) If $(1 - \alpha) \pi_B(c_A) < k$, there is a PBE under DP , at which B exits at the end of $t = 2$,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{2A}^{a*}, p_{2A}^{b*} \right), p_{2B}^{b*} \right\} &= \{(V_H, c_A), c_A\}, \\ \left(p_{tA}^*, p_{tA}^{b*} \right) &= (V_H, p_A^m) \text{ for } t = 3, \dots, T; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in $t = 1, 2$, and all consumers purchase from A in $t = 3, \dots, T$.

Proof. (i) If in $t = 1$ both firms follow the equilibrium prices and consumers make the purchases as described, their actions in $t = 2, \dots, T$, and only these actions, are sequentially rational, following the equilibrium belief that consumers with histories a and b are H - and L -consumers, respectively, while the off-equilibrium belief and price are $\left\{ \mu_{tj}^o = 1, p_{tj}^o = V_H \right\}$. It thus suffices to show that no player can profit from any deviation in $t = 1$.

First, given the prices on and off the equilibrium paths, each consumer's purchase in $t = 1$ is optimal. Second, if A deviates to a lower price that would attract some (and all) L -consumers, in order for the deviation to potentially benefit A , it must not reduce A 's profit for the period $t = 1$. Thus the lowest potentially profitable deviating price at $t = 1$ by A is \underline{p} . But since from (A2),

$$s(\underline{p}) - s(p_B^m) < \delta [s(c_A) - s(V_H)] = \delta s(c_A),$$

with each L -consumer believing that she will be offered V_H and c_A in $t = 2$ for purchasing from A and B in $t = 1$, respectively, no L -consumer will purchase from A at $t = 1$ for any deviating price A is willing to offer. Thus A cannot profit from any deviation at $t = 1$. Finally, it is obvious that B cannot benefit from changing its price at $t = 1$.

(ii) For any $t = 2, \dots, T$, if all H - and L -consumers purchased from A and B respectively in $t = 1$, B remains in the market in t , the unique equilibrium outcome in t is $(p_{tj}^*, p_{tj}^{b*}, p_{tj}^{o*}) = (V_H, c_A, V_H)$; and it follows that, since $(1 - \alpha)\pi_B(c_A) < k$, B must stay out of the market and A will charge $(p_{tA}^*, p_{tA}^{b*}, p_{tA}^{o*}) = (V_H, p_A^m, V_H)$ for $t = 3, \dots, T$. Thus, it suffices to show that there is an equilibrium in which, following the equilibrium outcome in the continuation game, $\{p_{1A}^*, p_{1B}^*\} = \{V_H, p_B^m\}$ and all H - and L -consumers purchase from A and B respectively in $t = 1$.

First, each consumer is optimizing in $t = 1$ given the firms' strategies. Next, A is willing to lower its price at most to \underline{p} in $t = 1$ in order to sell to all consumers in that period. But no L -consumer will purchase from A at such a price, with the expectation that she will be offered V_H and c_A in $t = 2$, respectively, for purchasing from A and B in $t = 1$, since

$$s(\underline{p}) - s(p_B^m) < \delta [s(c_A) - s(V_H)] = \delta s(c_A).$$

Hence A cannot benefit from any deviation in $t = 1$. Finally, B cannot benefit from changing its price at $t = 1$. *Q.E.D.*

We have not established equilibrium uniqueness in Proposition 3. Potentially, there might be equilibria where the (complete) separation of consumers does not occur in $t = 1$. For instance, we have not been able to rule out the possibility of an equilibrium where firm A prices slightly above \underline{p} in $t = 1$ and sells to all L -consumers, followed by A and B 's prices $\{V_H, p_B^m\}$ in $t = 2$, and price discrimination occurs afterwards. However, the equilibrium in Proposition 3 involves most price discrimination, in the sense that price discrimination both occurs at the earliest possible time and lasts the longest. We shall focus our analysis on this equilibrium.

We can compare the equilibrium outcomes under conditions (C2) and (C1). With uniform price the market is more competitive and equilibrium prices lower under (C2), while with discriminatory price the equilibrium outcome is the same under either (C2) and (C1). Thus, different from the basic model, price discrimination now raises price for the H -consumers in all periods. Furthermore, price discrimination also raises price for the L -consumers in $t = 1$ while lowers price for them in future periods; and the reason for the higher price in $t = 1$ is that L -consumers become less price sensitive in $t = 1$ under DP , because they take into account the future prices they will be offered, which depend on their current purchases.

The fact that equilibrium price under DP is not (weakly) lower than under UP for every period, even when there is no exit, makes the comparison of consumer welfare for the two pricing regimes more complicated. But unambiguous comparisons are available if both T and δ are sufficiently large.

Corollary 2. Assume Condition (C2) holds. If $(1 - \alpha) \pi_B(c_A) \geq k$, aggregate consumer surplus is higher under DP than under UP for all periods $t \geq 2$; otherwise, aggregate consumer surplus is lower under DP than under UP for all t except possibly $t = 2$. When T and δ are sufficiently large,

$$W^d \begin{cases} > W^u & \text{if } (1 - \alpha) \pi_B(c_A) \geq k \\ < W^u & \text{if } (1 - \alpha) \pi_B(c_A) < k \end{cases} .$$

Proof. (i) If $(1 - \alpha) \pi_B(c_A) \geq k$, firm B will remain in the market for all periods. Denote consumer welfare, industry profit, and total social surplus in period t by w_t^j , π_t^j , and z_t^j , for $j = d, u$. Then, for $t \geq 2$,

$$\begin{aligned}\pi_t^d &= [\alpha (V_H - c_A) + (1 - \alpha) \pi_B(c_A)], \\ \pi_t^u &= [\alpha (V_H - c_A) + (1 - \alpha) \pi_B(\underline{p})],\end{aligned}$$

and hence, $\pi_t^d - \pi_t^u = (1 - \alpha) [\pi_B(c_A) - \pi_B(\underline{p})] < 0$. Notice also that for $t \geq 2$, $z_t^d > z_t^u$ since output is higher under DP and expected unit cost is lower under DP . Therefore

$$w_t^d - w_t^u \equiv w^d - w^u > 0 \text{ for } t = 2, \dots, T.$$

Furthermore,

$$\begin{aligned}W^d - W^u &= w_1^d - w_1^u + (w^d - w^u) \delta (1 + \delta + \dots + \delta^{T-2}) \\ &\rightarrow w_1^d - w_1^u + (w^d - w^u) \frac{\delta}{1 - \delta} > 0 \text{ when } T \rightarrow \infty, \delta \rightarrow 1.\end{aligned}$$

(ii) If $(1 - \alpha) \pi_B(c_A) < k$, firm B will stay out of the market for $t \geq 3$.

$$\begin{aligned}w_t^u &= \alpha (V_H - p_A^u) + (1 - \alpha) \int s(p) dF_{\min}(p) > \alpha (V_H - p_A^u) + (1 - \alpha) \int s(p) dF_B(p) \\ &\geq \alpha (V_H - p_A^u) + (1 - \alpha) s(p_B^u) \quad \forall t,\end{aligned}$$

while $w_1^d = (1 - \alpha) s(p_B^m)$, $w_2^d = (1 - \alpha) s(c_A)$, and $w_t^d = (1 - \alpha) s(p_A^m)$ for $t > 2$. Thus,

$$w_t^d - w_t^u = (1 - \alpha) s(p_A^m) - [\alpha (V_H - p_A^u) + (1 - \alpha) s(p_B^u)] < 0 \text{ for } t > 2.$$

Furthermore, when $T \rightarrow \infty$, $\delta \rightarrow 1$,

$$\begin{aligned}W^d - W^u &\leq w_1^d - w_1^u + \delta [w_2^d - w_2^u] \\ &\quad + \delta^2 [(1 - \alpha) s(p_A^m) - [\alpha (V_H - p_A^u) + (1 - \alpha) s(p_B^u)]] (1 + \delta + \dots + \delta^{T-3}) \\ &\rightarrow w_1^d - w_1^u + \delta [w_2^d - w_2^u] + \delta^2 [(1 - \alpha) s(p_A^m) - [\alpha (V_H - p_A^u) + (1 - \alpha) s(p_B^u)]] \frac{1}{1 - \delta} < 0.\end{aligned}$$

Q.E.D.

Since increased competition, in the sense of lower p_A^u and p_B^u , increases consumer welfare under UP but does not change consumer welfare under DP , $W^d - W^u$ is lower with increased competition. In other words:

Remark 3. When the market is more competitive under uniform pricing, consumers gain less (or loss more) from price discrimination.

As in the basic model, the lowest price firm A charges is c_A . Thus in equilibrium firm A again does not engage in below-cost pricing.

4. WITH CONSUMER SWITCHING COST

We now extend the basic model in another direction: suppose that a consumer incurs switching cost $\sigma > 0$ each time she changes her supplier. We assume that $\sigma < s(p_A^m)$ (thus σ is not too large), and a stronger version of (C1) is satisfied:

$$\max_{c_A \leq p \leq \bar{p}} \{\alpha(p - c_A) + (1 - \alpha)\pi_A(p)\} < \alpha(V_H - c_A), \quad (\text{C1}')$$

where $\bar{p} \in (p_B^m, V_H)$ is defined by

$$s(\bar{p}) = s(p_B^m) - \sigma.$$

Notice that (C1') implies (C1). Condition (C1') will simplify our analysis but is not essential for the main insight of this section. Everything else is the same as in the basic model.

Under uniform price, the solution to the game is straightforward. In the last period, $t = T$, regardless of how the L -consumers purchase in $T - 1$, the unique Nash equilibrium is for A to charge V_H and for B to charge p_B^m , with all H - and L -consumers purchasing from A and B , respectively. Notice that by condition (C1'), A is better off charging V_H than charging any price not exceeding \bar{p} in T , and $s(p_B^m) - \sigma > s(p)$ for $p > \bar{p}$. From backward induction, we thus have:

Remark 4. In the presence of consumer switching cost, there is a unique SPNE under UP , where

$$\{p_{tA}^{u*}, p_{tB}^{u*}\} = \{V_H, p_B^m\} \text{ for } t = 1, 2, \dots, T,$$

all H -consumers purchase from A and all L -consumers purchase from B in all periods.

The equilibrium outcome here is the same as in the basic model of Section 2. Again, competition under UP is weak: there is no head-to-head competition between the two firms due to (C1').

Next we turn to the analysis under discriminatory price. Define $c_1 < c_2$ by the following conditions:

$$s(c_1) = s(c_A) + \sigma, \quad (5)$$

$$s(c_2) = s(c_A) - \sigma. \quad (6)$$

Then c_i exist uniquely and $0 < c_1 < c_A < c_2$, provided that c_A is not too small and σ not too large, which we shall assume.

Under DP , there are three possible cases for which the equilibrium analysis differs: (i) $k \leq (1 - \alpha)\pi_B(c_1)$, which implies that $c_1 > c_B$; (ii) $k > (1 - \alpha)\pi_B(c_2)$, and (iii) $(1 - \alpha)\pi_B(c_1) < k \leq (1 - \alpha)\pi_B(c_2)$. We first consider cases (i) and (ii). Define \tilde{p} by

$$s(\tilde{p}) - [s(c_A) - \sigma] = \delta [s(c_A) - s(c_2)] = \delta\sigma,$$

or $s(\tilde{p}) = s(c_A) - \sigma(1 - \delta)$. Then \tilde{p} exists uniquely and $c_A < \tilde{p} < c_2$.

Proposition 4. (i) If $k \leq (1 - \alpha)\pi_B(c_1)$, there is a unique PBE, where B always remains in the market,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{tA}^{a*}, p_{2A}^{b*} \right), \left(p_{tB}^{b*} \right) \right\} &= \{(V_H, c_A), \tilde{p}\} \text{ for } t = 2, \dots, T - 1, \\ \left\{ \left(p_{TA}^{a*}, p_{3A}^{b*} \right), \left(p_{TB}^{b*} \right) \right\} &= \{(V_H, c_A), c_2\}; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in all periods.

(ii) If $k > (1 - \alpha)\pi_B(c_2)$, there is a unique PBE, where B exits at the end of $t = 2$,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{2A}^{a*}, p_{2A}^{b*} \right), p_{2B}^{b*} \right\} &= \{(V_H, c_A), \tilde{p}\}, \\ \left(p_{tA}^{a*}, p_{tA}^{b*} \right) &= \{(V_H, p_A^m)\} \text{ for } t = 3, \dots, T; \end{aligned}$$

all H - and L -consumers purchase from A and B respectively in $t = 1, 2$, and all consumers purchase from A in $t = 3, \dots, T$.

Proof. See the Appendix.

We next turn to case (iii). Let $p_a < c_A$ and $p_b \leq c_B$ satisfy

$$\pi_A(p_a) + \delta\pi_A(p_A^m) = 0 \quad (7)$$

$$(1 - \alpha)\pi_B(p_b) + \delta[(1 - \alpha)\pi_B(c_2) - k] = 0. \quad (8)$$

Then p_a and p_b exist uniquely. Notice that both $c_A - p_a$ and $p_b - p_a$ increase as $\delta\pi_A(p_A^m)$ increases. Define

$$s(\hat{p}) = s(p_b) + \sigma(1 - \delta). \quad (9)$$

We note that there are parameter values under which $0 < p_a < \hat{p} < p_b$. It is always true that $\hat{p} < p_b$, and it is likely that $0 < p_a < \hat{p}$ if c_A is not too small and $c_A - c_B$ is not too large.

Proposition 5. Assume $(1 - \alpha)\pi_B(c_1) < k \leq (1 - \alpha)\pi_B(c_2)$. If in addition $0 < p_a < \hat{p}$, then there is a PBE at which firm B exits at the end of $t = 2$,

$$\begin{aligned} \{p_{1A}^*, p_{1B}^*\} &= \{V_H, p_B^m\}, \\ \left\{ \left(p_{2A}^{a*}, p_{2A}^{b*} \right), p_{2B}^{b*} \right\} &= \begin{cases} \{(V_H, \hat{p}), p_b\} & \text{if } T \text{ is an odd number} \\ \{(V_H, c_A), \tilde{p}\} & \text{if } T \text{ is an even number} \end{cases}, \\ \left(p_{tA}^{a*}, p_{tA}^{b*} \right) &= (V_H, p_A^m) \text{ for } t = 3, \dots, T; \end{aligned}$$

all H -consumers purchase from A in all periods; all L -consumers purchase from B in $t = 1$ and purchase from A for all $t > 2$; and in $t = 2$ all L -consumers purchase from A if T is an odd number but from B if T is an even number.

Proof. See the Appendix.

Notice that since $\hat{p} < p_b \leq c_B \leq c_A$, firm A may price below its average variable (marginal) cost in $t = 2$. Thus, with switching costs, it is possible that the dominant firm will engage in equilibrium below-cost pricing under DP , with the purpose and effect of eliminating competition. However, even with switching cost, the dominant firm may be able to drive

the smaller competitor from the market through dynamic price discrimination, without resorting to below-cost pricing.¹¹

Corollary 3. Assume $0 < p_a < \hat{p}$. In the presence of consumer switching costs, if $k \leq (1 - \alpha) \pi_B(c_1)$, DP increases consumer welfare; if $(1 - \alpha) \pi_B(c_1) < k$, DP may decrease consumer welfare, and A may engage in below-cost pricing with the purpose and effect of eliminating competition.

Proof. If $k \leq (1 - \alpha) \pi_B(c_1)$,

$$\begin{aligned} W^d &= (1 - \alpha) [s(p_B^m) + s(\tilde{p}) (\delta + \dots + \delta^{T-2}) + s(c_2) \delta^{T-1}], \\ W^u &= (1 - \alpha) [s(p_B^m) (1 + \delta + \dots + \delta^{T-1})]. \end{aligned}$$

Therefore

$$W^d - W^u = (1 - \alpha) \{ [s(\tilde{p}) - s(p_B^m)] (\delta + \dots + \delta^{T-2}) + [s(c_2) - s(p_B^m)] \delta^{T-1} \} > 0.$$

If $k > (1 - \alpha) \pi_B(c_2)$,

$$\begin{aligned} &W^d - W^u \\ &= (1 - \alpha) [s(p_B^m) + \delta s(\tilde{p}) + \delta^2 s(p_A^m) (1 + \delta + \dots + \delta^{T-3}) - s(p_B^m) (1 + \delta + \dots + \delta^{T-1})] \\ &= (1 - \alpha) \delta \{ s(\tilde{p}) - s(p_B^m) - \delta^2 [s(p_B^m) - s(p_A^m)] (1 + \delta + \dots + \delta^{T-3}) \} \\ &\rightarrow (1 - \alpha) \delta \left[s(\tilde{p}) - s(p_B^m) - \delta^2 [s(p_B^m) - s(p_A^m)] \frac{1}{1 - \delta} \right] < 0 \end{aligned}$$

when $c_A > c_B$, $T \rightarrow \infty$, and $\delta \rightarrow 1$.

If $(1 - \alpha) \pi_B(c_1) < k \leq (1 - \alpha) \pi_B(c_2)$, at the equilibrium characterized in Proposition

¹¹To characterize all possible equilibria for all possible parameter values appears to be a rather daunting task, and for the purpose of this paper we have not pursued in that direction.

5, A may price at $\hat{p} < c_A$ in $t = 2$ and

$$\begin{aligned}
& W^d - W^u \\
\leq & (1 - \alpha) [s(p_B^m) + \delta s(\hat{p}) + \delta^2 s(p_A^m) (1 + \delta + \dots + \delta^{T-3})] \\
& - (1 - \alpha) s(p_B^m) (1 + \delta + \dots + \delta^{T-1}) \\
= & (1 - \alpha) \delta \{ [s(\hat{p}) - s(p_B^m)] - \delta^2 [s(p_B^m) - s(p_A^m)] (1 + \delta + \dots + \delta^{T-3}) \} \\
\rightarrow & (1 - \alpha) \delta \left\{ [s(\hat{p}) - s(p_B^m)] - [s(p_B^m) - s(p_A^m)] \frac{\delta^2}{1 - \delta} \right\} < 0
\end{aligned}$$

when $c_A > c_B$, $T \rightarrow \infty$, and $\delta \rightarrow 1$. *Q.E.D.*

As in the basic model, DP benefits consumers if it does not cause the weaker firm to exit, due to intensified competition; but DP can harm consumers if it causes the weaker firm to exit.

Dynamic pricing based on purchase history occurs usually for two reasons. First, consumers may have inherent preference diversity for different firms' products and past purchases reveal such preferences. second, consumers may have costs to switch suppliers, which is another source of product differentiation that separates consumers under repeated purchases. Our analysis suggests that the nature of dynamic price discrimination in markets with asymmetric firms depends on whether or not switching cost is present. If DP is based purely on consumers' inherent brand preferences, below-cost pricing does not occur as an equilibrium strategy. However, in the presence of switching cost, consumers have the incentive to purchase from their current supplier, and a firm thus benefits from having a higher market share. Consequently, a dominant firm sometimes has the incentive to engage in "predatory" below-cost pricing. It loses in the short run for pricing below cost, but its low price deprives the rival of the market share needed to profitably remain in the market. The dominant firm is then able to increase its future prices and profits sufficiently to recoup the short-term losses.

5. CONCLUSION

The effects of dynamic price discrimination on competition and consumer welfare change dramatically from symmetric to asymmetric firms. The intensified competition under price discrimination, which tends to benefit consumers in markets with symmetric firms, can cause the exit of a weaker firm and harm consumers in markets with asymmetric firms. A sufficient condition for dynamic price discrimination to benefit consumers is that it does not result in fewer firms and that consumers have a long time horizon. When price discrimination induces exit, there may be a trade off for consumer welfare between current price reductions and future price increases, and price discrimination can reduce consumer welfare. The less efficient the dominant firm is, and/or the more competitive the market is under uniform price, the less (more) consumers tends to benefit (lose) from discriminatory price. Dynamic price discrimination may or may not appear as predatory in the usual sense, depending crucially on whether there is consumer switching cost: if dynamic pricing is based purely on consumer's brand preferences, it will not involve below-cost prices, even when it causes exit and harms consumers; in the presence of consumer switching costs, the dominant firm may price below cost in selling to the rival's customers, with the purpose and effect of eliminating competition.

Our analysis is based on variants of a duopoly model that are highly stylized, the choice of which is motivated by both analytical convenience and antitrust concerns. The equilibrium feature that only the stronger firm has the strict incentive to engage in purchase-history based price discrimination is clearly due to our simplifying assumption that the high-value consumers will only purchase from the stronger firm, but it also reflects the observation that it is the dominant firm who price discriminates in some actual antitrust cases involving asymmetric firms. That there may be market segmentation under uniform price, where the stronger and weaker firms sell to different segments of the market without head-to-head competition, simplifies the analysis and highlights the intensifying-competition effect of price discrimination; but it also seems plausible in reality, and Section 3 shows how this assumption can be relaxed. The assumption that the low-value consumers consider the

two firms' products as perfect substitutes is again for analytical convenience; it is possible to introduce some product differentiation for this group of consumers as well, but this substantially complicates the analysis, especially with many periods, due to the need by both firms to draw inferences about the types of this group of consumers based on past purchases.¹²

For antitrust policies, our analysis suggests that when a stronger firm engages in below-cost pricing that targets a weaker rival's customers, there is compelling reason to consider such pricing behavior as predatory and harming competition. However, even without below-cost pricing, dynamic price discrimination by a stronger firm can have an exclusionary purpose and effect; but it is also possible that the price discrimination benefits consumers. In such situations, the competitive effects can be determined only in the context of each case through detailed economic analysis. In particular, discriminatory price by the stronger firm would be anticompetitive when there is a "dangerous probability" that it will result in the competitor's exit, otherwise it is unlikely to raise significant antitrust concerns.

¹²In fact, instead of two type consumers, we could alternatively consider a Hotelling-type model where firms' asymmetry arises due to consumers' different transportation costs (preference intensities) towards the two firms, as in Chen (2006), or due to consumers' valuation differences from the two firms and/or from firms' cost differences. Such a model, however, appears much less tractable, with no clear advantage of offering additional insights.

REFERENCES

- [1] Armstrong, Mark (2005), "Recent Developments in the Economics of Price Discrimination", Mimeo, University College London.
- [2] Armstrong, Mark and John Vickers (1993), "Price Discrimination, Competition and Regulation", *Journal of Industrial Economics*, 41, 335-360.
- [3] Bolton, Patrick, Joseph Brodley, and Michael Riordan (2000), "Predatory Pricing: Strategic Theory and Legal Policy", *Georgetown Law Journal*, 88, 2239-2330.
- [4] Chen, Yongmin (1997), "Paying Customers to Switch", *Journal of Economics and Management Strategy* 6, 877-897
- [5] Chen, Yongmin (2005), "Oligopoly Price Discrimination by Purchase History" in *Pros and Cons of Price Discrimination*, Swedish Competition Authority: Stockholm.
- [6] Chen, Yongmin (2006), "Marketing Innovation", *Journal of Economics and Management Strategy*, 15, 101-123.
- [7] Corts, Kenneth (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment", *Rand Journal of Economics* 29, 306-323.
- [8] Farrell, Joseph and Carl Shapiro (1988), "Dynamic Competition with Switching Costs", *RAND Journal of Economics* 19, 123-137.
- [9] Farrell, Joseph and Paul Klemperer (2004), "Coordination and Lock-In: Competition with Switching Costs and Network Effects", in *Handbook of Industrial Organization*, Vol. 3, forthcoming.
- [10] Fudenberg, Drew and Jean Tirole (2000), "Customer Poaching and Brand Switching", *RAND Journal of Economics* 31, 634-657.
- [11] Fudenberg, Drew and J. Miguel Villas-Boas (2005), "Behavior-Based Price Discrimination and Customer Recognition", Mimeo.

- [12] Holmes, Thomas (1989), "The Effects of Third-Degree Price Discrimination in Oligopoly", *American Economic Review* 79, 244-250.
- [13] Klemperer, Paul (1987a), "The Competitiveness of Markets with Switching Costs", *RAND Journal of Economics* 18, 138-150.
- [14] Klemperer, Paul (1987b), "Entry Deterrence in Markets with Consumer Switching Costs", *Economic Journal*, 97, 99-117.
- [15] Stole, Lars (2004), "Price Discrimination and Imperfect Competition", *Handbook of Industrial Organization*, Vol. 3, forthcoming.
- [16] Taylor, Curtis (2003), "Supplier Surfing: Price Discrimination in Markets with Repeat Purchases". *RAND Journal of Economics* 34, 223-246.
- [17] Villas-Boas, J. Miguel (1999), "Dynamic Competition with Customer Recognition". *RAND Journal of Economics* 30, 604-631.

APPENDIX

The appendix contains the proofs for Propositions 4-5.

Proof of Proposition 4: (i) First, suppose that all players follow the proposed equilibrium actions in $t = 1$. Then, the only belief consistent with equilibrium is $\{\mu_{ij}^a, \mu_{ij}^b\} = \{1, 0\}$. Since there can be no beneficial deviations associated with the H -consumers, we only need to consider the prices for the L -consumers. If the L -consumers purchase from B in $t = T - 1$ at price $p_{T-1} = \tilde{p}$, in $t = T$ the equilibrium prices must be c_2 for B and c_A for A , with each L -consumer receiving surplus $s(c_2)$ purchasing from B ; if L -consumers purchase from A in $t = T - 1$, in $t = T$ the equilibrium prices must be c_1 for B and c_A for A , with each L -consumer receiving surplus $s(c_A)$ purchasing from B . Either way in equilibrium B remains in market and L -consumers purchase from B in $t = T$. Therefore A must offer c_A to the L -consumers in $t = T - 1, T$, and, since

$$s(p_{T-1}) - [s(c_A) - \sigma] = \delta [s(c_A) - s(c_2)],$$

it is optimal for B to offer $p_{T-1} = \tilde{p}$ to the L -consumers in $t = T - 1$, provided that the L -consumers have purchased from B in $T - 2$.

Next, suppose that L -consumer have purchased from B in $t = T - 3$. All L -consumer will purchases from B in $T - 2$ at p_{T-2} if

$$s(p_{T-2}) + \delta s(p_{T-1}) + \delta^2 s(c_2) = s(c_A) - \sigma + \delta s(c_A) + \delta^2 s(c_A),$$

or

$$s(p_{T-2}) = s(c_A) - \sigma(1 - \delta) = \tilde{p},$$

where we notice that if an L -consumer purchases from A in $T-2$, she will receive $\delta [s(c_A) + \delta s(c_A)]$ in $T - 1$ and T , either by staying with A or switching back to B , due to the bidding by A and B and the fact that A will bid c_A . Furthermore, if

$$p_{T-k} = s(c_A) - \sigma(1 - \delta) \text{ for } k = 1, \dots, T - 2,$$

we have

$$\begin{aligned} & s(p_{T-(k+1)}) + \delta s(p_{T-k}) + \delta^2 s(p_{T-k+1}) + \dots + \delta^k s(p_{T-1}) + \delta^{k+1} s(c_2) \\ = & s(c_A) - \sigma + \delta s(c_A) + \delta^2 s(c_A) + \dots + \delta^{k+1} s(c_A), \end{aligned}$$

or

$$s(p_{T-(k+1)}) + [s(c_A) - \sigma(1 - \delta)](\delta + \dots + \delta^k) + \delta^{k+1}[s(c_A) - \sigma] = s(c_A)(1 + \delta + \dots + \delta^k) - \sigma,$$

or $s(p_{T-(k+1)}) = s(c_A) - \sigma(1 - \delta) = \tilde{p}$. It follows that if all L -consumers purchase from B in $t = 1$, it is uniquely sequentially rational for A to offer c_A , B to offer \tilde{p} for $t = 2, \dots, T$ to the L -consumers, under belief $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$, and for all L -consumers to purchase from B in all periods. We have thus shown that, if all players follow the proposed equilibrium actions in $t = 1$, then their actions in the following periods in the proposed equilibrium, and only these actions, are sequentially rational under equilibrium belief $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$ and off equilibrium belief $\mu_{tj}^o = 1$. Moreover, there can be no other equilibrium belief consistent with the proposed strategies in $t = 1$.

Next, in $t = 1$, due to condition (C1'), A cannot benefit from choosing any price different from V_H , given B 's price and consumers' purchasing decisions; and B 's price is obviously optimal. Under firms' prices and under belief $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$ and off equilibrium belief $\mu_{tj}^o = 1$, consumers' purchasing strategies are also optimal. We have thus shown the proposed is indeed a PBE.

Finally, if there is any other PBE, it must involve some $\{\tilde{p}_{1A}^*, \tilde{p}_{1B}^*\} \neq \{V_H, p_B^m\}$. but then either A would benefit from deviating to V_H , or B would benefit from deviating to p_B^m , or both would benefit from the deviations. Thus the PBE is unique.

(ii) First, again suppose that all players follow the proposed equilibrium actions in $t = 1$. Then, the only belief consistent with equilibrium is $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$. In $t = T$, the most B can charge the L -consumers and still sell to them is c_2 . Since $k > (1 - \alpha)\pi_B(c_2)$, B will not be in the market for $t = T$. By backward induction, B must exit in the end of $t = 2$, which means that A must charge L -consumers c_A in $t = 2$ and p_A^m in $t = 3, \dots, T$, while B

charges the L -consumers \tilde{p} in $t = 2$ so that

$$s(\tilde{p}) + \delta [s(p_A^m) - \sigma] = s(c_A) - \sigma + \delta s(p_A^m).$$

Therefore, if all players follow the proposed equilibrium actions in $t = 1$, then their actions in the following periods in the proposed equilibrium, and only these actions, are sequentially rational under equilibrium belief $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$ and off equilibrium belief $\mu_{tj}^o = 1$; and there can be no other equilibrium belief consistent with the proposed strategies in $t = 1$.

As in (i) above, the proposed actions in $t = 1$ are optimal for each player, and thus the proposed is a PBE; and moreover the PBE is unique.

Q.E.D.

Proof of Proposition 5. First, along the equilibrium path, if all L consumers purchase from A at $t = T - 1$, firm B will exit and A 's profit from L -consumers is $(1 - \alpha) \pi_A(p_A^m)$ at $t = T$; if all L -consumers purchase from B at $t = T - 1$, firm B will stay in the market at $t = T$ to earn $(1 - \alpha) \pi_B(c_2) - k$ with A earning 0 from the L -consumers. Thus at $t = T - 1$ to sell to the L -consumers firm A is willing to charge them as low as p_a , and firm B is willing to sell to the L -consumers at a price as low as p_b if it is already in the market.

Next, if L -consumers purchase from A in $T - 2$, since $p_a < p_b < c_B$, B will not be in the market for $t = T - 1$ if it needs to incur k . If L -consumers purchase from B in $T - 2$, since by assumption $p_a < \hat{p}$, and since

$$s(\hat{p}) - \sigma + \delta s(p_A^m) = s(p_b) + \delta [s(p_A^m) - \sigma],$$

or $s(\hat{p}) = s(p_b) + (1 - \delta) \sigma$, it is optimal for each L -consumer to purchase from A at a price slightly below \hat{p} instead of from B at p_b ; thus it is also optimal for B to stay out of the market in $T - 1$. Therefore, regardless of from whom L -consumers purchase in $T - 2$, it is sequentially rational for B to stay out of the market in $T - 1$ and T .

Next, if L -consumers purchase from A in $T - 3$, since $(1 - \alpha) \pi_B(c_1) < k$, B will not be in the market for $t = T - 2$ if it needs to incur k . If L -consumers purchased from B in $T - 3$, B will be in the market for $T - 2$, with A and B 's price to the L -consumers being c_A and c_2 , respectively. From the same reasoning of backward induction, we have:

If L -consumers purchase from A in $T - 4$, B will not be in the market for $t = T - 3$ if it needs to incur k . If L -consumers purchase from B in $T - 4$, A and B 's price to the L -consumers will be \hat{p} and p_b respectively in $T - 3$, and thus B will also not be in the market for $T - 3$ if it needs to incur k .

If L -consumers purchase from A in $T - 5$, B will not be in the market for $t = T - 4$ if it needs to incur k ; but if L -consumers purchase from B in $T - 5$, B will be in the market for $T - 4$ even if it needs to incur k .

Regardless of from whom L -consumers purchased in $T - 6$, B will not be in the market for $T - 5$ if it needs to incur k

Therefore, if $T = 3$, given the actions of all parties at $t = 1$ at the proposed equilibrium, the actions in the following periods are sequentially rational given the equilibrium belief $\{\mu_{tj}^a, \mu_{tj}^b\} = \{1, 0\}$ and off equilibrium belief $\mu_{tj}^o = 1$.

If $T = 4$, the same is true as when $T = 3$. In particular, now $p_{2A}^{b*} = c_A$ and $p_{2B}^{b*} = \tilde{p}$, with $s(\tilde{p}) = s(c_A) - \sigma(1 - \delta)$.

If $T = 5$, the same is also true as when $T = 3$. in particular, $p_{2A}^{b*} = \hat{p}$ and $p_{2B}^{b*} = p_b$, with

$$s(\hat{p}) - \sigma + \delta s(p_A^m) + \delta^2 s(p_A^m) = s(p_b) + \delta [s(p_A^m) - \sigma] + \delta^2 s(p_A^m).$$

Similarly for $T = 6, T = 7, \dots$

To establish the PBE, it remains to show that no player can benefit from unilateral deviation in $t = 1$. For A , any deviation will reduce its profit in $t = 1$ without increasing its future profit. The same is true for B . And obviously no consumer can benefit from deviation, given the firms' beliefs and prices.

Q.E.D.