The Risk of Contagion from Multimarket Contact

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Abstract

It is conventional wisdom that the strategic linkage of markets that is enabled by multimarket contact typically increases the profitability of cooperation among rivals. We find to the contrary that a strong force against strategic linkage results from imperfect monitoring of adherence to cooperation. With such imperfections, strategically linking markets can lower payoffs by permitting the impact of adverse shocks in one market to spread to others. Consequently, players of repeated games on more than one front may find it strictly advantageous to avoid linking strategies on a front with clear monitoring to outcomes on a front with error-prone monitoring. One implication is that antitrust, competitive strategy, and foreign policy analyses have presumed too broadly that multimarket contact fosters cooperation. The game-theoretic equilibria characterized here shed light on why players such as firms and nations sometimes strategically link fronts in their rivalry, and sometimes take care to articulate that some fronts of particularly volatile conflicting interests will not trigger broader adverse moves against the rival.

1 Introduction

"Multimarket contact" refers to a strategic situation in which rivals interact on multiple fronts, and each rival can link its strategies so that outcomes on one front influence actions on the others. For nearly 50 years economists have argued that each rival's ability to link its strategies across fronts where there is multimarket contact will typically be profitably exploited.[1] It has become conventional wisdom that permitting retaliation on all fronts to be triggered by a rival's aggressive action on just one front usually allows more profitable equilibria to be supported, because the prospect of the more severe punishment can overcome a rival's short-term incentive to deviate from mutually profitable cooperation.[2]

This paper shows that the conventional wisdom on the general advantages from strategic linkages across fronts does not hold up in the presence of realistic asymmetric information about rivals' actions. In settings where participants' actions are monitored with unavoidable noise and error, repeated-game equilibria typically entail the possibility of mistaken retaliatory punishment when rivals are erroneously perceived to have deviated from cooperation. In such settings, we show that strategic linkages can be disastrously injurious because they allow mistaken retaliatory punishment to spread from one front to another. The losses due to this contagion can readily outweigh any gains from strategic linkage. We identify conditions under which strategic linkages are disadvantageous due to multimarket contagion, as well as conditions under which strategic linkages are profitable for the players.

Better understanding of the impact of strategic linkages across fronts is important in a number of domains. Concern about the anticompetitive effects of multimarket contact has prominently influenced recent antitrust analysis and litigation. For example, under the Sherman Act the U.S. government sued the major airlines and their venture Airline Tariff Publishing Co. ("ATP") for using ATP's computer network to communicate and negotiate privately about pricing moves and about actual and threatened retaliatory moves in different markets.[3] In *U.S. v. Omnipoint*,

[1] See Corwin Edwards, as quoted in Bernheim and Whinston [1990].

[2] The seminal work of Bernheim and Whinston [1990] shows that this articulation of the conventional wisdom is over-simplified, because the extra retaliation might be balanced by the extra gain from deviating simultaneously on all fronts. They show that there is no advantage from strategic linkage where the different fronts are proportional to each other in their impacts on payoffs from deviation and from punishment. Where this proportionality is absent, and where complete cooperation on all fronts cannot otherwise be sustained, they show that strategic linkages may improve equilibrium payoffs.

[3] As reported in the *Wall Street Journal* ("Fare Warning: How Airlines Trade Price Plans", Oct. 9, 1990), "Mr. Elkins testified that Northwest preferred not to lower fares in the Chicago market, dominated by UAL Corp.'s United and AMR Corp.'s American, because those rivals were sure to retaliate with cheap prices in Minneapolis, with 'devastating' consequences."

the U.S. government sued the mobile telephone service supplier Omnipoint for using the trailing digits of its bids in an auction for spectrum licenses to signal strategic linkages among its and rivals' bidding conduct in several markets. Several opponents of the merger between the Union Pacific and the Southern Pacific Railroads argued that the merged entity would have little trouble sustaining anticompetitive price coordination with the other large railroad, the Burlington Northern/Santa Fe, because the merger would create substantial multimarket contact between the two firms. In general, the extent of multimarket contact has become a significant factor in assessing postmerger coordinated competitive effects.

Moving beyond the industrial organization context, in his seminal work, Schelling [1960] discusses the merits of countries' linking negotiations on political and economic fronts. More recent work by Ederington [2001, 2002, 2003] considers the linkage of trade and domestic policies in bilateral relationships that in some instances is permitted under the GATT. Maggi [1999] examines an alternative interpretation of linkage in which the World Trade Organization provides information to third parties that enables multilateral responses to violations of bilateral agreements.

The informal argument that multimarket contact enhances coordination obtained crucial theoretical support from Bernheim and Whinston [1990]. They showed that strategically linking markets increases profits because it relaxes the incentive constraints that limit firms' ability to sustain collusive behavior in settings of repeated competition. Their technical result coincides with the intuition that multimarket contact permits linkage-induced punishment that can deter local deviations from collusive behavior. For present purposes, it is important to realize that linkage is never harmful in the framework of Bernheim and Whinston [1990], in the sense that any level of profits sustained without linking markets can also be sustained by linking them. Of course, in many instances strategically linking markets strictly increases profits.

Several empirical studies have provided evidence that suggests that multimarket contact strictly increases profits. Experimental studies have found that subjects transfer market power from less competitive to more competitive markets, which is one of the theory's predictions.[4] Field studies have found that prices are higher in a particular geographic market the more frequently the competing firms meet in other markets.[5]

Despite the insights provided by prior theoretical and empirical results, failure to consider salient

[4] See Feinberg and Sherman [1985, 1988] and Phillips and Mason [1992].

[5] See Evans and Kessides [1994], Barla [1994], and Singal [1996] (airlines); Jans and Rosenbaum [1996] (cement); Parker and Roller [1994] (cellular phones).

features of real strategic environments may contribute to a misunderstanding of when linkage is desirable. To address this potential shortcoming, we introduce into the competitive environment the realistic assumption that players are uncertain about what actions their rivals have taken. In a well known paper, Green and Porter [1984] show that such noise hinders collusion in a single market because adverse shocks must be followed by punishment in order to deter deviation from collusive behavior. The means by which unobservability affects collusion in a single market suggests that strategic linkage may promote a contagion that allows adverse shocks to spread from one market to another. To explore this possibility, we completely characterize the effect on collusion of the strategic linkage of two fronts, one in which rivals' actions are observed perfectly and one in which they are not. While we find in some instances that strategic linkage increases payoffs, in others we find that it reduces payoffs because destabilizing shocks spread from one front to the other. Every instance in which linkage is harmful is one for which the intuition from perfect information models suggests that it should be helpful. This result occurs because linkage actually does enable previously unsustainable collusive behavior, but the cost of sustaining this behavior outweighs the benefits, in terms of payoffs lost through the contagious launch of more severe linkage-induced punishment. This cost is not a factor in settings with perfect information, because the punishment at issue is never invoked in equilibrium.

Two other papers examine the effect of imperfect monitoring and multimarket contact, albeit in different forms from the one evaluated in this paper. Matsushima [2001] uses a different structure for market noise and a somewhat different underlying duopoly model to show that nearly efficient collusion can be sustained through the linkage of a sufficiently large number of identical markets. Considering a large number of markets effectively eliminates uncertainty by using the Law of Large Numbers to almost certainly detect deviations. However, in many instances there may not be so many markets available. If one modifies Matushima's framework by considering only a small number of asymmetric markets, then using our methodology our central result emerges that collusion sustained through linkage can reduce payoffs. Ederington [2003], in the setting of his that is most similar to ours, finds the stark result that linkage always reduces payoffs. However, his model has important structural differences from ours. In particular, he does not consider separate fronts, but instead considers two imperfectly substitutable actions that both influence a single outcome. Moreover, he only considers collusive schemes that entail excessively harsh punishment for apparent deviations. Because of the structural differences between our two approaches, it is not clear if his result holds when considering optimal collusive schemes of the sort that we evaluate in Section 3.

Because rivals often can choose not to link markets in which doing so would be harmful, we are not suggesting that the mere ability to link leads to worse outcomes in the situations that we identify. However, for several reasons we believe that our results provide useful insights about the desirability of using linkage to sustain collusion. First, we have a complete set of results that characterize when multimarket contact can be profitably exploited, and we find that the intuition from perfect information models overstates the extent to which multimarket contact is helpful. Second, in practice there appears to be a widely held view that actual linkage typically strictly increases payoffs. We show that this view is incorrect in settings where there are possible inaccuracies in monitoring rivals' actions. Moreover, we show that such monitoring problems also overturn the more refined view, based on Bernheim and Whinston [1990] and Ederington [2002], that any payoff that can be achieved without linkage also can be achieved with linkage. Third, there may exist situations in which linkage is difficult to avoid. For example, the nature of the data provided by supermarket scanners makes it difficult for a producer to know its rivals' couponing activity, even though its rivals' shelf prices can be observed. This leads to a structural linkage of the available strategic instruments, because each producer's payoff from setting certain prices also depends on its and its rivals' unobserved couponing activity. In political arenas, decision makers may be unwilling to separate fronts for reasons unrelated to the immediate strategic interaction. In theaters of military rivalry, definitional lines between geographic areas and between categories of weapons may be sufficiently unclear to make problematic the strategic separation of different fronts.

Our results have potentially significant implications for antitrust policy. First, informational factors that suggest that coordination is difficult to sustain may also indicate that strategically linking markets will be disadvantageous to the firms. In such instances, the mere existence of multimarket contact between rival firms should not be viewed as reliable evidence that the involved markets are conducive to coordinated competitive effects. Second, our modeling approach illuminates what should be seen as the essential features of the recent antitrust cases that turn on multimarket contact. In both the ATP and Omnipoint cases, institutional characteristics enabled the firms to communicate and to observe each other's pricing with relative accuracy. The ATP computer network was accessed by all the major airlines and enabled them accurately to observe each others' new prices and to visibly respond, before the prices were available to consumers. The spectrum auction that was the context for Omnipoint's challenged conduct permitted the openly observable use of trailing digits of bids for accurate communications by the bidders in early rounds of the iterated auction, leaving an ample number of subsequent rounds for reactions and implementation of agreements. These market settings had characteristics that inoculated the firms' conduct against the contagion we identify, and as a consequence the multimarket contact aided their profitable coordination of pricing.

Our results also have implications for international relations, where in some areas a misunderstanding of multimarket contact's efficacy would be particularly dangerous. For example, during the height of the Cold War, imagine if thermonuclear strikes were strategically linked to fence line shootings at Guantanamo Bay. It is frequently the case that nations deliberately avoid strategic linkages that would result in too much danger from the consequent destabilization of otherwise mutually gainful cooperation, but input from academic advisers that linkage never hurts and typically helps may lead to dangerous policy choices.[6] This may be particularly relevant for the practice of brinkmanship, which in essence is the linking of fronts with possibly very small probabilities. In all of the instances for which we find that strategically linking fronts strictly lowers payoffs, our result holds even for arbitrarily small chances that outcomes on one front will influence actions on the other.

Finally, our results have implications for empirical strategies for determining the effectiveness of multimarket contact in increasing payoffs. It is possible that multimarket contact has an even larger effect than has been identified in existing field studies, because there may exist uncontrolled variation in the data in terms of whether or not linkage is actually helpful. That is, if in some instances linkage is helpful and is used, while in other instances linkage is not helpful and is not used, then the true effect of linkage in the helpful instances will be underestimated if relevant control variables are omitted. The same insight could be used to evaluate the effect of treaties that are careful to define the extent of linkage permitted in between-country interactions.

In the next section we introduce a pair of infinitely repeated prisoners' dilemma games - a "safe front" with accurate observations of payoffs and past actions, and a "risky front" subject to random adverse shocks that make rivals' actions indistinguishable from those that selfishly destroy cooperation. Throughout Section 2 we analyze these games under the simplifying assumption that the players are restricted to using grim strategies that punish deviation from cooperation with perpetual play of the stage game Nash equilibrium. When the adverse shocks are sufficiently likely

[6] As Ederington [2002] describes, conflicting policies in different areas illustrate that in the WTO there is no clear consensus on the desirability of linking trade and domestic policies.

to make it impossible to sustain cooperation on the risky front alone, it may nevertheless be possible to sustain cooperation on both fronts by strategically linking the grim punishments to deviation on either front. The surprising result uncovered in Section 2 is that it may be disadvantageous to sustain cooperation through linkage. This will be the case if the linkage creates a relatively large expected loss of the benefits of cooperation on the safe front, through contamination by an adverse shock on the risky front.

In Section 3 we enrich the analysis of the same pair of repeated games by completely characterizing the effects of multimarket contact on the ability and incentives of players to collude when using "optimal" punishment strategies. These strategies avoid the grim strategies' excessive punishment by allowing more complex intertemporal patterns of punishment. Once again we find that strategic linkage lowers profits for a wide range of cases, despite the fact that strategic linkage can stabilize the risky front to an extent sufficient to permit cooperative behavior on both fronts. Unlike with grim strategies, here the key condition for the profitability of linkage does not turn on the relative size of the expected loss of the benefits of cooperation on the safe front. Instead, the key condition characterizes whether the risky front is worth stabilizing by any sufficient punishment, in view of the likelihood that the punishment will be triggered by the adverse shocks. Section 4 briefly concludes, while an Appendix contains all proofs.

2 Enforcing Collusion Using Grim Strategies

Throughout the paper, we consider an infinitely repeated two-player game, in each period of which players 1 and 2 simultaneously select an action for each of the prisoners' dilemmas presented in Figure 1. We refer to these two games as the risky front and the safe front, respectively. On the risky front, in each period both players receive z_r with probability θ, regardless of their respective actions on that front. We refer to this event as an adverse or negative shock. With probability $1-\theta$, both players receive the payoffs associated with the combination of their respective actions on the risky front. We refer to this event as a positive shock. We assume that the shocks on the risky front are independently and identically distributed over time. On the safe front, in each period both players receive the payoffs associated with the combination of their respective actions on that front. On either front, the players do not see the action chosen by their rival. Rather, they know only their own action and their own payoff. They can infer their rival's action from their own payoff on the safe front. However, on the risky front, the rival's action cannot be inferred accurately from one's own payoff, due to the possibility of a negative shock that leads to the same payoff as that forthcoming from the rival playing D.

We restrict the payoffs to make the stage games prisoners' dilemmas by assuming $z_r < w_r < x_r$ and $y_s < z_s < w_s < x_s$. In addition, we assume that $\frac{z_r+x_r}{2} < w_r$ and that $\frac{y_s+x_s}{2} < w_s$ to ensure that the sum of the players' payoffs is maximized by mutual cooperation.

Figure 1

	C	D
C	w_r, w_r	z_r, x_r
D	x_r, z_r	z_r, z_r

Risky Front

	C	D
C	w_s, w_s	y_s, x_s
D	x_s, y_s	z_s, z_s

Safe Front

We use a slightly nonstandard payoff structure on the risky front to prevent the players from having an avenue through which they can avoid the uncertainty associated with negative shocks. Specifically, if the payoff from selecting C when the rival selected D differed from the payoff from selecting D when the rival selected D, and if negative shocks yielded a payoff equal to that from when the rival deviated from mutual selection of C, then it might be more profitable for the players to attempt to collude by alternating between (D, C) and (C, D). By doing so, negative shocks would be apparent for what they are, and could not be confused with defection by one's rival. Despite this modification, our formulation of the payoff structure affords a great deal of flexibility. For example, by varying the payoffs we can make one front "bigger" or more important than the other, and we can create differences across fronts in the relative payoff from collusion versus defection.

In the supergame the players may use **unlinked strategies** that treat the two fronts independently, or they may use **linked strategies** for which the action specified on one front depends upon prior outcomes on the other front. We refer to the associated equilibria as being unlinked or linked, respectively.

In the next two subsections we analyze the collusive payoffs associated with unlinked and linked strategies under the assumption that players are restricted to using grim trigger strategies. These strategies have the feature that the players cooperate by selecting C in each period until a specified event triggers perpetual reversion to selecting D in each period.

2.1 Grim Collusion on One Front

If players 1 and 2 choose their actions on each front in each period according to unlinked strategies, then we can solve separately for the maximal collusive payoffs on the two fronts. We first analyze the maximal collusive payoff on the safe front, which is a fairly standard exercise. According to the grim strategy, collusion is sustained by both players selecting C in each period until a specified event triggers perpetual reversion to selecting D in each period. Let $\overline{V}_s^g$ denote the net present value of the maximal collusive payoff using grim strategies on the safe front, where the subscript "s" refers to the safe front and the superscript "g" refers to the grim strategy. We wish to determine the value of $\overline{V}_s^g$, which can be written as

$$\overline{V}_s^g = w_s + \delta V^c. \tag{1}$$

δ is the discount factor common to both players, and V^c is the net present value of play starting next period, following the mutual selection of the collusive action C in the current period. Equation (1) states that the net present value of collusion can be decomposed into the current payoff from collusion today and the discounted net present value of play starting tomorrow, following collusion today. Because this is a grim strategy environment, V^c can take on only one of two values: either continue with collusion, so $V^c = \overline{V}_s^g$, or revert to perpetual selection of D, so $V^c = \frac{z_s}{1-\delta}$.

For collusion to be sustained, it must be the case that

$$w_s + \delta V^c \geq x_s + \delta V^p, \tag{2}$$

where V^p is the net present value of play starting next period, following selection of D in the current period. Equation (2) is an incentive constraint requiring that the payoff from collusion exceeds the payoff from deviating today by selecting D and then earning the punishment payoff V^p starting tomorrow. As with the value of V^c, V^p can take on only one of two values: either $V^p = \overline{V}_s^g$, or $V^p = \frac{z_s}{1-\delta}$.

In order to determine the value of $\overline{V}_s^g$, note that $\overline{V}_s^g$ is increased and the incentive constraint is relaxed by selecting $V^c = \overline{V}_s^g$. Additionally, $\overline{V}_s^g$ is unaffected and the incentive constraint is relaxed by selecting $V^p = \frac{z_s}{1-\delta}$. Thus, collusive behavior in the current period is followed by continued collusive behavior, while defection is met with reversion to perpetual mutual selection of D. Using these two preliminary results in equations (1) and (2), we find that

$$\overline{V}_s^g = \frac{w_s}{1-\delta},$$

provided that

$$\delta \left(x_s - z_s \right) \geq x_s - w_s,$$

or equivalently that

$$\left(\frac{\delta}{1-\delta} \right) \left(w_s - z_s \right) \geq x_s - w_s.$$

If the preceding constraint does not hold, then collusion cannot be sustained, in which case both players select D each period, and $\overline{V}_s^g = \frac{z_s}{1-\delta}$. Thus, the players can collude on the safe front provided that the short-term gain from defection, $x_s - w_s$, is less than the long-term loss from defection. This loss is given by the discounted net present value of the difference between the forever-repeated collusive and punishment payoffs, $\left(\frac{\delta}{1-\delta} \right) \left(w_s - z_s \right)$. Notice that because $x_s > w_s > z_s$, there always exists a discount factor such that collusion can be sustained.

We now analyze the maximal collusive payoff on the risky front, whose unseen volatility makes for a slightly more difficult problem than on the safe front. As on the safe front, one can show that maximally collusive behavior features both players selecting C in each period until there is an apparent defection. If both players select C, then both receive w_r with probability $(1-\theta)$ and receive z_r with probability θ. However, if one player deviates by selecting D when C is specified, then that player receives x_r with probability $(1-\theta)$. The other player earns z_r with probability 1 and is faced with an inference problem. Namely, did he earn z_r because of a negative shock, or because his rival deviated by selecting D?

This inference problem creates the following complication. After the outcome of play in a particular period, each player must determine what action to choose in the next period. If both players selected C and there was a positive shock, then both players know that neither player deviated. Hence, the players will want to continue their collusive play. However, if player 1 deviated by selecting D, then he knows that player 2 received z_r but that player 2 is unsure why. Therefore, player 2's choice of action in the next period cannot depend on what player 1 chose in the current period, but instead depends upon a set of possible actions chosen by player 1. Player 1 is aware of player 2's dilemma, and therefore his choice next period also cannot depend upon his choice in the current period. Similar reasoning applies if player 2 deviated by selecting D in the current period, or if both selected C but there was a negative shock.

To be more technical, given the structure of uncertainty, there are only two common knowledge events following the simultaneous selection of actions in a period in which C was to be selected: either both players received at least w_r, or at least one player did not. That is, the finest partition of the information sets in Table 1 on which the players agree is {{E},{F,G,H}}. For example, if the shock is positive and player 2 deviates by selecting D, then the players both could not say that they are in state G. Player 1 is unsure whether he received z_r because player 2 selected D, state G, or because the shock was negative, state H. Given that there are only two common knowledge events following play in a collusive period, there are at most two strategy profiles that can be employed from the next period onward.

Table 1

	2 received w_r or x_r	2 received z_r
1 received w_r or x_r	E	F
1 received z_r	G	H

We wish to maximize the collusive payoff on the risky front, $\overline{V}_r^g$, which can be written as

$$\overline{V}_r^g = (1-\theta)\left[w_r + \delta \overline{V}_r^g\right] + \theta\left[z_r + \delta V^p\right]. \tag{3}$$

Equation (3) states that the net present value of collusion can be decomposed into two components, depending on whether or not there is a positive shock. The first component can be further decomposed into the payoff today from collusion when there is a positive shock and the net present value of collusion starting tomorrow. Similarly, the second component can be decomposed into the payoff today from collusion when there is a negative shock and the net present value of play starting tomorrow following an apparent defection today.

For collusion to be sustained, it must be the case that

$$(1-\theta)\left[w_r + \delta \overline{V}_r^g\right] + \theta\left[z_r + \delta V^p\right] \geq (1-\theta)\left[x_r + \delta V^p\right] + \theta\left[z_r + \delta V^p\right]. \tag{4}$$

Equation (4) is an incentive constraint requiring that the expected payoff from colluding exceeds the expected payoff from deviating today by selecting D and then earning the punishment payoff V^p starting tomorrow. As this is a grim strategy environment, V^p can take on only one of two values: an apparent defection is followed either by permanent reversion to mutual selection of D, so $V^p = \frac{z_r}{1-\delta}$, or by continued cooperative play, so $V^p = \overline{V}_r^g$. The latter may be reasonable if θ is very large, as in that case bad shocks are so likely that one should not view z_r as evidence that the rival has cheated. However, one can easily show that if $V^p = \overline{V}_r^g$, then equation (4) cannot be satisfied. That is, for collusion on the risky front to be sustained, apparent defection must be punished. Using these preliminary results in equations (3) and (4), we find

$$\overline{V}_r^g = \frac{(1-\theta)\, w_r + \theta z_r + \frac{\theta\delta z_r}{1-\delta}}{1-(1-\theta)\delta},$$

provided that

$$(1-\theta)\delta\,(x_r - z_r) \geq x_r - w_r,$$

or equivalently that

$$\left(\frac{(1-\theta)\delta}{1-(1-\theta)\delta}\right)(w_r - z_r) \geq x_r - w_r.$$

If the preceding constraint does not hold, then collusion cannot be sustained, in which case both players select D in each period, and $\overline{V}_r^g = \frac{z_r}{1-\delta}$. Otherwise collusion can be sustained, but it is only partial in the sense that cooperation will give way to perpetual mutual play of D following the first adverse shock. Thus, the players can partially collude on the risky front provided that the short-term gain from defection, $x_r - w_r$, is less than the expected long-term loss from defection. This loss is given by the expected discounted net present value of the difference between the forever-repeated collusive and punishment payoffs, $\left(\frac{(1-\theta)\delta}{1-(1-\theta)\delta}\right)(w_r - z_r)$, where the discount factor is modified to account for the probability that there is no adverse shock. The greater the likelihood of adverse shocks the more difficult it is to sustain collusion. In contrast to the safe front, there exist θ for which collusion cannot be sustained for any δ. Thus, the risky front's volatility plays a critical role in the sustainability of collusion, a point to which we will return later.

2.2 Grim Collusion on Multiple Fronts

The analysis in subsection 2.1 assumes that the players attempt to collude using unlinked strategies for which the actions on one front are independent of prior outcomes on the other front. Having learned from the analysis in Bernheim and Whinston [1990] that multimarket contact enhances cooperation, the players might consider linking a front that has perfect information with a front that has imperfect information. For example, in an industrial organization context, price and quantity information might be easy to monitor, while advertising or R&D outlays might be very difficult to monitor. Firms may see strategically linking prices and advertising expenditures as their best opportunity to increase profits, as opposed to linking R&D spending and advertising expenditures. From an international relations perspective, adherence to a policy of nuclear nonproliferation may be easier to monitor than adherence to a policy of nonaggression by submarines in the North Atlantic. Similarly, trade policies may be much more transparent than domestic policies that influence trade.

Suppose that players 1 and 2 choose their actions on each front in each period according to linked strategies. As when using unlinked strategies on the risky front, there are only two common knowledge events following a period in which C was to be selected on the risky front: either both players received at least w_r on the risky front, or at least one player did not. Linkage with the safe front does not create any additional issues regarding common knowledge, because it is common knowledge what actions are taken on the safe front.

The maximal collusive payoff on the two fronts, $\overline{V}^g_{rs}$, can be written as

$$\overline{V}^g_{rs} = (1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right]. \tag{5}$$

Equation (5) states that the payoff from collusion can be decomposed into two parts, depending on whether there is a positive or negative shock on the risky front. Each part can be further decomposed into the short-term payoff from selecting the collusive action on each front and the long-term payoff for each common knowledge contingency. We wish to maximize $\overline{V}^g_{rs}$ with respect to V^c and V^p_r, subject to

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[x_r + w_s + \delta V^p_r\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \tag{6}$$

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[w_r + x_s + \delta V^p_s\right] + \theta\left[z_r + x_s + \delta V^p_{rs}\right] \tag{7}$$

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[x_r + x_s + \delta V^p_{rs}\right] + \theta\left[z_r + x_s + \delta V^p_{rs}\right] \tag{8}$$

Equations (6) through (8) are incentive constraints requiring that the expected payoff from collusion exceeds the expected payoff from deviating on the risky front only, the safe front only, and both fronts, respectively.

If $V^p_r = V^c$, then one can show that equation (6) cannot hold. Therefore, an apparent defection on the risky front only is met with the mutual selection of D forever, so $V^p_r = \frac{z_s+z_r}{1-\delta}$. Decreasing either V^p_s or V^p_{rs} relaxes (7) and (8) without affecting (6) or the value of the objective function. Therefore, V^p_s and V^p_{rs} also should be set to the lowest individually rational level, $\frac{z_s+z_r}{1-\delta}$. With the preceding results, one can easily show that if equation (8) holds, then equations (6) and (7) also hold. That is, if defection on both fronts is deterred, then defection on only one front is also deterred. Thus, equation (8) is the only incentive constraint that is relevant for determining $\overline{V}^g_{rs}$. Increasing V^c relaxes (6)-(8) and increases the value of the objective function. Therefore,

$V^c = \overline{V}^g_{rs}$, and successful collusion is followed by continued collusion. Using the preceding results regarding V^c, V^p_r, V^p_s, and V^p_{rs} in equations (5) and (8), we find

$$\overline{V}^g_{rs} = \frac{w_s + \frac{\theta\delta z_s}{1-\delta} + (1-\theta)\, w_r + \theta z_r + \frac{\theta\delta z_r}{1-\delta}}{1-(1-\theta)\delta},$$

provided that

$$\delta\,(x_s - z_s) + (1-\theta)\delta\,(x_r - z_r) \geq \frac{x_s - w_s}{1-\theta} + x_r - w_r,$$

or equivalently that

$$\left(\frac{(1-\theta)\delta}{1-(1-\theta)\delta}\right)\left[(w_s - z_s) + (1-\theta)\,(w_r - z_r)\right] \geq (x_s - w_s) + (1-\theta)\,(x_r - w_r)\,.$$

Otherwise, collusion on both fronts cannot be sustained through linkage.

While several parameter-dependent conclusions emerge from our analysis of multimarket contact using grim strategies, we focus on the two that we find most interesting. First, if collusion can be sustained on both fronts by using unlinked strategies, then linking the two fronts strictly reduces the players' payoffs. In this instance, full collusion is possible on the safe front, while partial collusion is possible on the risky front. Linkage does not permit more collusive behavior on the risky front than was possible in the unlinked equilibrium, as the players already were selecting C in each period until there was an apparent deviation. Hence, the only effect of the linkage is to permit the shocks from the risky front to spread to the safe front and destroy the perpetual collusion that was possible there. In fact, collusion on both fronts may not be sustainable with linked strategies even though collusion could be sustained on both fronts by using unlinked strategies. The reason is that the potential for adverse shocks to spread to the safe front and prompt punishment despite the players' adherence to the collusive scheme creates a greater incentive to defect on the safe front. For example, if defection on the safe front were just barely prevented using unlinked strategies, then the additional payoff risk imposed by linkage could make collusion on the safe front unsustainable.

Second, if collusion can be sustained on the safe front but not on the risky front in the unlinked equilibrium, then collusion on both fronts can possibly be sustained through linkage. This is unsurprising because the safe front's incentive constraint is satisfied in the unlinked equilibrium, which means there exists excess enforcement power. If the safe front's incentive constraint is sufficiently slack, then the incentive constraint in the linked equilibrium also can be satisfied. However, comparing a player's total payoffs in the linked and unlinked equilibria yields the surprising result that if collusion can be sustained through linkage, then it strictly reduces the players' payoffs if $(1-\theta)(w_r - z_r) < \theta\left(\frac{\delta}{1-\delta}\right)(w_s - z_s)$. The expression on the left hand side is the expected gain from colluding for one period on the risky front, while the expression on the right hand side is the expected loss from colluding for one period on the risky front; namely, if there is a negative shock in the current period, then all future collusive gains on the safe front are lost. As in the case we considered in the preceding paragraph, linkage reduces the players' payoffs because it permits the negative shocks to spread from the risky front to the safe front. Comparing the expected gains and losses from sustaining collusion on the risky front through linkage reveals that linkage is more likely to reduce the players' payoffs the more likely is a negative shock (θ), the smaller is the per-period gain from collusion versus mutual defection on the risky front ($w_r - z_r$), the larger is the per-period gain from collusion versus mutual defection on the safe front ($w_s - z_s$), and the more the players care about the future (δ).

The results in this section show that multimarket contact can strictly increase payoffs by transferring enforcement power from one front to the other. However, collusion that can be sustained through multimarket contact may strictly reduce the players' payoffs because shocks can spread and destabilize collusion that would otherwise be sustainable. In such a situation, the players are strictly better off not coordinating their strategies across the two fronts.

3 Enforcing Collusion Using Optimal Strategies

Although the grim strategies employed in Section 2 are simple, familiar, and frequently used in analyses of cooperation, in the setting we study here the punishment they specify is more than sufficient to deter deviation. While excessive punishment is not invoked in equilibrium when players have perfect information, it is when they do not. Consequently, sustaining collusion by using excessive punishment reduces the players' payoffs more than is necessary.

In this section we enrich our analysis in Section 2 by solving for the highest payoff associated with a symmetric sequential equilibrium (SSE).[7] SSEs have the feature that after the outcome in a period, the net present values of the players' payoffs from the next period onward also are implemented through SSEs. We refer to the strategies developed by Abreu, Pearce, and Stacchetti [1986,1990] (hereinafter APS) that achieve the highest SSE payoff as **optimal** strategies. The key benefit of using optimal strategies in this setting is flexibility, because the continuation payoffs can take on any value between the net present value of the individually rational payoff and the maximal

[7] The restriction to symmetric equilibria is natural given the symmetric nature of the problem. See Abreu, Pearce, and Stacchetti [1990] for a discussion of asymmetric equilibria.

collusive payoff. In contrast, the continuation payoffs when using grim strategies are limited to only two values, the net present value of the individually rational payoff and the maximal collusive payoff. While using grim strategies is sufficient to show that there exist instances in which multimarket contact strictly increases payoffs, we must consider optimal strategies in order to illustrate the robustness of our earlier result that strategic linkage can strictly decrease the players' payoffs.

Before presenting our analysis, we briefly describe the APS methodology. Their primary insight is that a repeated game can be decomposed into a family of static games, much like a dynamic programming problem. Consider play following the first period of an SSE. The SSE specifies successor SSEs to be followed in each state of the world after the first period. These successor SSEs have associated payoffs. If the truncated game for each state of the world following the first period is simply replaced by the payoff to the associated successor SSE, then this new game's equilibrium is exactly the first period behavior specified by the original SSE. This approach is analogous to using backward induction to determine the subgame perfect Nash equilibrium in a finite extensive-form game. The APS methodology permits one to concentrate on payoffs rather than strategies, which is useful because the strategies potentially are quite complicated.

3.1 Optimal Collusion on One Front

As in Subsection 2.1, assume that the players sustain collusion by employing unlinked strategies. We first consider optimal collusion on the safe front. Let $\mathcal{V}_s \in \Re$ denote the set of payoffs of all SSEs for the game under consideration, where the subscript "s" refers to the safe front. $\mathcal{V}_s$ is nonempty (selecting D forever and earning z_s each period is an SSE) and is compact.[8] Thus, $\mathcal{V}_s$ has a largest element, $\overline{V}_s^o$, where the superscript "o" refers to the optimal strategy. $\overline{V}_s^o$ is associated with an SSE we denote $\overline{\sigma}_s$. We assume that the players are able to coordinate their actions by using a public randomization device, so that any payoff between the maximum and minimum SSE payoff can be achieved. While this assumption is used implicitly in determining the continuation payoffs following the first period of play, for expositional ease we assume that the collusive scheme begins by both players selecting C.[9]

Following first period play, $\overline{\sigma}_s$ specifies two SSEs truncated to the remaining game. Call these

[8] $\mathcal{V}_s$ clearly is bounded, and a limit argument shows that it is closed.

[9] For completeness' sake, Lemma 1 in the Appendix analyzes optimal collusion on both fronts assuming that players also may use the public randomization device in the first period of play in order to specify different combinations of actions across the two fronts. Similar results emerge in analyzing unlinked strategies. The results for the linked equilibrium have the knife-edge property that the players either want to select C on both fronts with probability 1, or want to select C only on one front. As we are interested in the effect of using linkage to sustain collusion on both fronts, here we focus on the former case.

truncated SSEs σ^p and σ^c, where the superscripts "p" and "c" serve as mnemonics for punishment and collusion, respectively. These truncated SSEs have associated payoffs V^p and V^c, which both are elements of $\mathcal{V}_s$. In solving for the maximal payoff, we concentrate on the payoffs associated with σ^p and σ^c rather than the strategies themselves. The payoff to $\overline{\sigma}_s$ is

$$\overline{V}_s^o = w_s + \delta V^c. \tag{9}$$

Equation (9) states that the net present value of collusion can be decomposed into the current payoff from collusion today and the net present value of play starting tomorrow, following successful collusion today. We wish to maximize $\overline{V}_s^o$ with respect to V^c, subject to the constraint

$$w_s + \delta V^c \geq x_s + \delta V^p. \tag{10}$$

Equation (10) is an incentive constraint requiring that the payoff from colluding must weakly exceed the payoff from defecting.

There are two immediate conclusions from the form of the maximization problem. First, because increasing V^c increases the objective function and relaxes (10), $V^c = \overline{V}_s^o$. Intuitively, if the players receive the collusive payoff in the current period, then they continue with the same behavior in the next period. Second, because V^p does not enter the objective function and relaxes (10) when decreased, $V^p = \frac{z_s}{1-\delta}$, the lowest individually rational level. Using these two preliminary results, we can rewrite the problem to maximize

$$\overline{V}_s^o = \frac{w_s}{1-\delta}, \tag{11}$$

subject to

$$\left(\frac{\delta}{1-\delta}\right)(w_s - z_s) \geq x_s - w_s. \tag{12}$$

As we found in the analysis of grim strategies, constraint (12) may be written equivalently as $\delta\,(x_s - z_s) \geq x_s - w_s$. The following proposition summarizes the results.

Proposition 1 *Collusion cannot be sustained on the safe front if and only if the short-term gain from defection exceeds the long-term loss. Formally, if $\delta\,(x_s - z_s) < x_s - w_s$, then collusion cannot be sustained, with*

$$\overline{V}_s^o = \frac{z_s}{1-\delta}.$$

If $\delta(x_s - z_s) \geq x_s - w_s$, then collusion can be sustained, with

$$\overline{V}_s^o = \frac{w_s}{1-\delta} > \frac{z_s}{1-\delta}.$$

Notice that both the maximal collusive payoff and the condition necessary for it to exceed the payoff from perpetual play of the static Nash equilibrium strategies are identical when using grim and optimal strategies. This is the case because grim strategies are optimal when the players' actions are perfectly observed, which follows because there is no noise in the competitive environment to accidentally trigger punishment.

We now consider optimal collusion on the risky front. Let $\mathcal{V}_r \in \Re$ denote the non-empty and compact set of payoffs of all SSEs for the game under consideration, where the subscript "r" refers to the risky front. Let $\overline{V}_r^o$ denote the largest element of $\mathcal{V}_r$, which is associated with an SSE we denote $\overline{\sigma}_r$. We assume that the collusive scheme begins by both players selecting C. If the players both select C, then they receive w_r with probability $(1-\theta)$ and receive z_r with probability θ. Moreover, there are only two common knowledge events following the simultaneous selection of actions in the period, just as we found in the analysis of collusion with grim strategies.

Given the two common knowledge events following first period play, $\overline{\sigma}_r$ specifies two SSEs truncated to the remaining game, σ^p and σ^c. These truncated SSEs have associated payoffs V^p and V^c, which both are elements of $\mathcal{V}_r$. The payoff to $\overline{\sigma}_r$ is

$$\overline{V}_r^o = (1-\theta)\left[w_r + \delta V^c\right] + \theta\left[z_r + \delta V^p\right]. \tag{13}$$

Equation (13) states that the expected payoff from collusion can be decomposed into two elements, depending on whether or not there was a positive or negative shock on the risky front. We wish to maximize $\overline{V}_r^o$ with respect to V^p and V^c, subject to the constraint

$$(1-\theta)\left[w_r + \delta V^c\right] + \theta\left[z_r + \delta V^p\right] \geq (1-\theta)\left[x_r + \delta V^p\right] + \theta\left[z_r + \delta V^p\right]. \tag{14}$$

Equation (14) is an incentive constraint requiring that the expected payoff from colluding must weakly exceed the expected payoff from defecting.

As in our analysis of the safe front, it is clear that $V^c = \overline{V}_r^o$. Moreover, the incentive constraint (14) must bind. If it did not, then one could increase V^p, still satisfy (14), yet increase $\overline{V}_r^o$. That is, the players should not punish an apparent deviation any more than is necessary to prevent an actual deviation, because the front's payoff-variability ensures that play occasionally will depart from the mutual selection of C, even though no player ever defects in equilibrium. This flexibility in selecting the continuation payoff following an apparent deviation is the key feature distinguishing optimal strategies from grim strategies. Using these two preliminary results, we can rewrite the problem to maximize

$$\overline{V}_r^o = \frac{w_r - \theta\left(x_r - z_r\right)}{1-\delta}, \tag{15}$$

subject to

$$\delta V^p = \delta \overline{V}_r^o - \left(x_r - w_r\right). \tag{16}$$

Moreover, individual rationality requires that $V^p \geq \frac{z_r}{1-\delta}$. The two constraints can be combined to yield the requirement that

$$\left(\frac{(1-\theta)\delta}{1-(1-\theta)\delta}\right)\left(w_r - z_r\right) \geq x_r - w_r,$$

which can be written equivalently as $(1-\theta)\delta\left(x_r - z_r\right) \geq x_r - w_r$. The following proposition summarizes the results.

Proposition 2 *Collusion cannot be sustained on the risky front if and only if the expected short-term gain from defection exceeds the expected long-term loss. If collusion is sustainable, then the expected collusive payoff exceeds the payoff from perpetual selection of D. Formally, if $(1-\theta)\delta\left(x_r - z_r\right) < x_r - w_r$, then collusion cannot be sustained, with*

$$\overline{V}_r^o = \frac{z_r}{1-\delta}.$$

If $(1-\theta)\delta\left(x_r - z_r\right) \geq x_r - w_r$, then collusion can be sustained, with

$$\overline{V}_r^o = \frac{w_r - \theta\left(x_r - z_r\right)}{1-\delta} > \frac{z_r}{1-\delta}$$

and

$$V^p = \frac{(1-\theta)\delta x_r + \theta\delta z_r - \left(x_r - w_r\right)}{\delta\left(1-\delta\right)}.$$

Notice that the constraint necessary for collusion to be sustainable is identical when using grim and optimal strategies. In fact, both strategies yield the same maximal collusive payoff when the constraint just binds, because in both instances the continuation payoff following an apparent deviation will be $\frac{z_r}{1-\delta}$. However, the maximal payoff from using optimal strategies strictly exceeds the maximal payoff from using grim strategies when the constraint is slack, because the punishment strategies are of optimal severity.

3.2 Optimal Collusion on Multiple Fronts

If the players employ linked strategies, then the punishment for deviation on a single front can be made more severe by reducing payoffs on the other front. To the extent that this more severe punishment both prevents deviation on the first front and is credible to implement, then we will see that strategic linkage enhances the players' ability to collude.

Let $\mathcal{V}_{rs} \in \Re$ denote the non-empty and compact set of all SSE payoffs when using linked strategies, where the subscript "rs" denotes the linkage of the risky and safe fronts, and let $\overline{\sigma}_{rs}$ denote the SSE that yields the largest element of $\mathcal{V}_{rs}$, $\overline{V}^o_{rs}$. In the first period of play, $\overline{\sigma}_{rs}$ specifies that each player selects C on each front. As in the preceding analysis, when selecting C on the risky front, in equilibrium there are only two common knowledge events following the choice of actions in the first period: either both players received at least w_r on the risky front, or at least one player did not. $\overline{\sigma}_{rs}$ specifies two SSEs truncated to the remaining game, one for each common knowledge contingency. These two SSEs are denoted σ^c and σ^p_r, with associated payoffs V^c and V^p_r. The SSEs specified by $\overline{\sigma}_{rs}$ following a defection on the safe front or on both the risky and safe fronts are denoted σ^p_s and σ^p_{rs}. These SSEs come into play only out of equilibrium, and they have associated payoffs V^p_s and V^p_{rs}. The expected payoff to $\overline{\sigma}_{rs}$ is

$$\overline{V}^o_{rs} = (1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right]. \tag{17}$$

We wish to maximize $\overline{V}^o_{rs}$ with respect to V^p_r, V^p_s, V^p_{rs}, and V^c, subject to the following constraints.

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[x_r + w_s + \delta V^p_r\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \tag{18}$$

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[w_r + x_s + \delta V^p_s\right] + \theta\left[z_r + x_s + \delta V^p_{rs}\right] \tag{19}$$

$$(1-\theta)\left[w_r + w_s + \delta V^c\right] + \theta\left[z_r + w_s + \delta V^p_r\right] \geq (1-\theta)\left[x_r + x_s + \delta V^p_{rs}\right] + \theta\left[z_r + x_s + \delta V^p_{rs}\right] \tag{20}$$

Equations (18)-(20) are incentive constraints requiring that the expected payoff from colluding must weakly exceed the expected payoff from deviating on the risky front only, on the safe front only, or on both fronts, respectively.

By familiar reasoning, we can show that $V^c = \overline{V}^o_{rs}$. Moreover, because decreasing either V^p_s

or V_{rs}^p does not affect the objective function but relaxes the incentive constraints, it must be the case that $V_s^p = V_{rs}^p = \frac{z_r+z_s}{1-\delta}$, the lowest individually rational level. Thus, perfect information about the players' actions on the safe front permits very severe punishment without fear of mistakenly invoking it. Using these two preliminary results, we can rewrite the problem to maximize

$$\overline{V}_{rs}^o = \frac{w_s + (1-\theta)w_r + \theta z_r + \theta\delta V_r^p}{1-(1-\theta)\delta}, \tag{21}$$

subject to the following constraints.

$$\begin{aligned} \delta\left[\overline{V}_{rs}^o - V_r^p\right] &\geq x_r - w_r & (22)\\ (1-\theta)\delta\left[\overline{V}_{rs}^o - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V_r^p - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s & (23)\\ (1-\theta)\delta\left[\overline{V}_{rs}^o - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V_r^p - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s + (1-\theta)(x_r - w_r) & (24) \end{aligned}$$

Constraints (22)-(24) are updated versions of constraints (18)-(20). It is clear that if (24) holds, then (23) holds. That is, if defection on both fronts is deterred, then defection on the safe front only is deterred. Therefore, the only relevant incentive constraints are (22) and (24). Finally, (22) must bind. If it did not, then one could increase V_r^p, relax (24), still satisfy (22), yet increase $\overline{V}_{rs}^o$. That is, the punishment for an apparent deviation on the risky front only should not be more severe than is necessary to prevent an actual deviation. With these intermediate results, the maximal collusive payoff from linking the safe and risky fronts is found by maximizing

$$\overline{V}_{rs}^o = \frac{w_s + (1-\theta)w_r + \theta z_r + \theta\delta V_r^p}{1-(1-\theta)\delta}, \tag{25}$$

with respect to V_r^p, subject to the following incentive constraints.

$$\begin{aligned} \delta\left[\overline{V}_{rs}^o - V_r^p\right] &= x_r - w_r & (26)\\ (1-\theta)\delta\left[\overline{V}_{rs}^o - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V_r^p - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s + (1-\theta)(x_r - w_r) & (27) \end{aligned}$$

Moreover, individual rationality requires that $V_r^p \geq \frac{z_r+z_s}{1-\delta}$.

Our analysis of the outcomes in the optimal linked equilibrium can be characterized by the four possible outcomes in the optimal unlinked equilibrium, collusion being sustainable or not on each of the two fronts. We consider each possibility separately.

Proposition 3 *Suppose that collusion cannot be sustained on either front by using optimal unlinked strategies. Collusion on both fronts can never be sustained through linkage. Formally, if* $(1-\theta)\,\delta\,(x_r - z_r) < x_r - w_r$ *and* $\delta\,(x_s - z_s) < x_s - w_s$*, then collusion on both fronts cannot be sustained through linkage, with*

$$\overline{V}^o_{rs} = \frac{z_r + z_s}{1-\delta} = \overline{V}^o_r + \overline{V}^o_s.$$

This result is not too surprising, because neither front has excess enforcement power that can be used to increase collusion on the other front. That is, collusion on one front cannot be supported by threatening more severe punishment on the other front, because the payoff on the other front is already at the lowest individually rational level. Therefore, collusion on both fronts cannot be sustained through linkage, which implies that multimarket contact does not change the players' payoffs. This outcome is identical to the outcome with grim strategies.

Proposition 4 *Suppose that collusion can be sustained on both fronts by using optimal unlinked strategies. Collusion on both fronts can always be sustained through linkage, but it leaves the players' payoffs unchanged. Formally, if* $(1-\theta)\,\delta\,(x_r - z_r) \geq x_r - w_r$ *and* $\delta\,(x_s - z_s) \geq x_s - w_s$*, then collusion on both fronts can be sustained through linkage, with*

$$\overline{V}^o_{rs} = \frac{w_r - \theta\,(x_r - z_r)}{1-\delta} + \frac{w_s}{1-\delta} = \overline{V}^o_r + \overline{V}^o_s$$

and

$$V^p_r = \frac{(1-\theta)\delta x_r + \theta\delta z_r - (x_r - w_r)}{\delta(1-\delta)} + \frac{w_s}{1-\delta}.$$

At first glance, one might argue that collusion is not sustained through linkage in this instance, because the continuation payoff following an apparent deviation on the risky front can be implemented by using the strategies from the optimal unlinked equilibrium. However, the continuation payoff can also be implemented by using a symmetric punishment scheme in which each player selects D on both fronts for a specified number of periods before returning to collusive play. In addition, the fronts are linked following any deviation on the safe front, because after such a deviation play on both fronts reverts to perpetual selection of D, regardless of the outcome on the risky front.

Readers familiar with the analysis and intuition from Bernheim and Whinston [1990] may find it surprising that the players' payoffs do not increase in this case, because the safe front's slack constraint indicates that there exists excess punishment power. By linking the two fronts it seemingly should be possible to increase the players' payoffs by increasing their payoffs on the risky front that in the unlinked equilibrium are limited by the unobservability of rivals' actions. However, the flaw in this reasoning is that linkage does not change the probability of entering the punishment phase that is associated with defection on the risky front. Because the players are not deviating on the risky front in either the unlinked or the linked equilibrium, punishment occurs according to the probability of a negative shock, which is identical in both cases. Manipulating the various continuation payoffs cannot be used to increase the players' payoffs. For example, the payoff following an apparent defection on the risky front cannot be increased with an associated decrease in the payoff following apparent defections on both the risky and safe fronts, because this will induce players to defect only on the risky front in every period. The more severe punishment never will be invoked, because the safe front never has a negative shock. Reducing the payoff following an apparent defection on the risky front, say by switching to less profitable behavior on the safe front, does not reduce defection on the risky front, because there already was no defection in equilibrium. Reducing that payoff merely reduces the overall payoff, because the punishment phase is entered only when there is a negative shock. If players already are not cheating, and only enter punishment phases to ensure that they do not cheat, then the safe front's excess enforcement power has no place to be applied.

This result also illustrates an important qualitative difference between the outcomes using grim and optimal strategies. If collusion is sustainable on both fronts in the unlinked equilibrium, then collusion sustained by using linked grim strategies strictly reduces the players' payoffs because it puts at risk the entire collusive payoff on the safe front. In contrast, with optimal strategies one can fine-tune the punishment to be no harsher than is necessary to sustain collusion, with the end result that linking is not harmful. Moreover, collusion quite possibly cannot be sustained through linkage by using grim strategies, even if it could be sustained by using unlinked strategies. This occurs because putting the safe front's entire collusive payoff at risk through linkage may make it impossible to prevent deviation on the safe front. With optimal strategies, collusion can always be sustained through linkage in this case.

Proposition 5 *Suppose that collusion can be sustained on the safe front but not on the risky front by using optimal unlinked strategies. Collusion on both fronts can sometimes be sustained through linkage, but if so it may strictly reduce the players' payoffs. Formally, suppose that* $\delta(x_s - z_s) \geq x_s - w_s$ *and* $(1-\theta)\delta(x_r - z_r) < x_r - w_r$*. If* $\delta(x_s - z_s) + (1-\theta)\delta(x_r - z_r) < (x_s - w_s) + (x_r - w_r)$*, then collusion on both fronts cannot be sustained through linkage. If* $\delta(x_s - z_s) + (1-\theta)\delta(x_r - z_r) \geq$

$(x_s - w_s) + (x_r - w_r)$, then collusion on both fronts can be sustained through linkage, with

$$\overline{V}^o_{rs} = \frac{w_r - \theta\left(x_r - z_r\right)}{1-\delta} + \frac{w_s}{1-\delta}$$

and

$$V^p_r = \frac{(1-\theta)\delta x_r + \theta\delta z_r - (x_r - w_r)}{\delta(1-\delta)} + \frac{w_s}{1-\delta}.$$

However, if $z_r > w_r - \theta\left(x_r - z_r\right)$, then collusion on both fronts sustained through linkage leads to strictly lower payoffs than collusion only on the safe front sustained by using unlinked strategies.

In this case, collusion on both fronts that is sustained through linkage may lead to strictly lower payoffs than from colluding on the safe front and abandoning collusion on the risky front, because the destabilizing nature of the risky front's shocks so adversely affects the safe front. This strange result occurs even though linkage enables previously unsustainable collusion on the risky front. That is, if $\delta\left(x_s - z_s\right) + (1-\theta)\,\delta\left(x_r - z_r\right) \geq \left(x_s - w_s\right) + \left(x_r - w_r\right)$ and $z_r > w_r - \theta\left(x_r - z_r\right)$, then the players can coordinate on C on each front through linkage, but the amount they gain by selecting C on the risky front cannot compensate for the loss so frequently incurred on the safe front.

To better understand the condition for unprofitable but sustainable collusion, first consider the unlinked equilibrium. If collusion is not sustainable on the risky front in the unlinked equilibrium, then that means that there is no individually rational punishment that deters deviation from collusive play. By strategically linking the risky front to the safe front, more severe punishments become individually rational, and these punishments may be severe enough to deter deviation from collusive play on the risky front. Specifically, an apparent deviation on the risky front can be followed by the most severe punishment available on the risky front, combined with a departure from collusive play on the safe front. To see that the collusive equilibrium specified in Proposition 5 exhibits such behavior, note that V^p_r is less than its value in the unlinked equilibrium, $\frac{z_r + w_s}{1-\delta}$, which is the payoff associated with perpetual selection of D on the risky front and perpetual selection of C on the safe front. Whether or not this punishment is individually rational is embodied in the pooled constraint that must be satisfied in order for collusion on both fronts to be sustainable by using linked strategies. The individual rationality of the punishment depends in part on how much slack is in the safe front's incentive constraint in the unlinked equilibrium.

However, the individually rational punishment described in Proposition 5 is not necessarily profitable to employ. Because that punishment will eventually be invoked by an adverse shock on the risky front, then the payoff from colluding using that punishment could be lower than the payoff associated with the strategies in the unlinked equilibrium. In particular, one can show that if $z_r > w_r - \theta(x_r - z_r)$, then the least harsh punishment necessary to sustain collusion on both fronts by using linked strategies leads to a total payoff less than that associated with using the optimal unlinked strategies. This condition is irrelevant in the unlinked equilibrium, because such a punishment is not individually rational (or even feasible) and so could not be implemented. However, such a punishment may be individually rational when using linked strategies.

We should point out that the finding that collusion sustained through linkage can be unprofitable holds even for arbitrarily small amounts of collusion on the risky front. That is, if $z_r > w_r - \theta(x_r - z_r)$, then any attempt to sustain collusive play on the risky front, no matter how infrequently, leads to strictly lower payoffs than if no collusive play were attempted there. We show this formally in Lemma 1 in the Appendix.

Proposition 5 also illustrates another difference between grim and optimal strategies. In this case the condition for which collusion sustained through linked grim strategies is unprofitable includes payoffs from both the safe front and the risky front. The comparable condition when collusion is sustained through linked optimal strategies does not include payoffs from the safe front. The difference emerges because linked grim strategies put the entire collusive payoff on the safe front at risk. Linked optimal strategies use only as much punishment as is necessary to prevent deviation on the risky front, and the necessary punishment does not depend on payoffs on the safe front. However, those losses still are incurred on the safe front, as a result of adverse shocks on the risky front.

The following two examples illustrate the effects of multimarket contact so far discussed, including the magnitude of the discrepancy between the intuition from perfect information models and the additional effects of imperfect monitoring on the profitability of sustaining collusion through linkage. The first example is illustrated by Figure 1, where we suppose that $w_r = w_s = 30$, $x_r = x_s = 50$, $y_s = 0$, and $z_r = z_s = 10$. If there is perfect monitoring of the rival's action on both fronts ($\theta = 0$), then the "irrelevance result" from Bernheim and Whinston [1990] emerges because the two fronts have identical payoffs. That is, collusion can be sustained in the unlinked equilibrium either on both fronts or on neither front, so strategic linkage yields no increase in the players' payoffs. Despite this result, there is a role for strategic linkage with imperfect monitoring on the risky front ($\theta > 0$). Regions IV and V, the horn shaped area below the constraint for the risky front and above the pooled constraint, comprise the set of (θ, δ) pairs for which collusion is sustainable on the safe front but not on the risky front in the unlinked equilibrium, and for which collusion on both fronts can be sustained through linkage. As determined in Proposition 5, however, collusion sustained through linkage is unprofitable in Region V, for which θ strictly exceeds 0.5. Region V accounts for approximately half of the entire set of (θ, δ) pairs for which collusion can be sustained through linkage, which indicates that imperfect monitoring can impose a significant constraint on multimarket contact's value in increasing the payoff from collusive behavior.

Figure 1 Here

The second example is illustrated by Figure 2, where we suppose that $w_r = 20, w_s = 30$, $x_r = 60, x_s = 50$, $y_s = 0$, and $z_r = z_s = 10$. If there is perfect monitoring of the rival's action on both fronts ($\theta = 0$), then for $\delta \in (0.67, 0.8)$ the strategic linkage of the safe and risky fronts enables otherwise unsustainable collusion on the risky front. This is precisely the story from Bernheim and Whinston [1990], in which asymmetries across fronts provide a means for strategic linkage to play a role. Consider a value of δ such as 0.78. As θ increases from 0, strategic linkage continues to allow collusion to be sustained on the risky front. This collusion is profitable until $\theta = 0.2$. For a range of θ beyond this point, collusion on both fronts still can be sustained, but that collusion is unprofitable due to the contagious effects of the negative shocks on the risky front. The negative effects of imperfect monitoring are even larger than in the first example, because sustainable collusion is now unprofitable for well over half of the entire set of (θ, δ) pairs for which collusion can be sustained through linkage.

Figure 2 Here

Proposition 6 *Suppose that collusion can be sustained on the risky front but not on the safe front by using optimal unlinked strategies. Collusion on both fronts can sometimes be sustained through linkage, and if so it strictly increases the players' payoffs. Formally, suppose* $(1-\theta)\,\delta\,(x_r - z_r) \geq x_r - w_r$ *and* $\delta\,(x_s - z_s) < x_s - w_s$. *If* $\delta\,(x_s - z_s) + (1-\theta)\,\delta\,(x_r - z_r) < (x_s - w_s) + (x_r - w_r)$, *then collusion on both fronts cannot be sustained through linkage. If* $\delta\,(x_s - z_s) + (1-\theta)\,\delta\,(x_r - z_r) \geq (x_s - w_s) + (x_r - w_r)$, *then collusion on both fronts can be sustained through linkage, with*

$$\overline{V}^o_{rs} = \frac{w_r - \theta\,(x_r - z_r)}{1-\delta} + \frac{w_s}{1-\delta} > \overline{V}^o_r + \overline{V}^o_s$$

and

$$V_r^p = \frac{(1-\theta)\delta x_r + \theta\delta z_r - (x_r - w_r)}{\delta(1-\delta)} + \frac{w_s}{1-\delta}.$$

In Proposition 6 the risky front is used to sustain collusion on the safe front, an approach that at first glance is counterintuitive. The players can now coordinate on (C, C) on the safe front, because if they cheat there they also will lose their entire collusive payoff on the risky front. Sustaining collusion through linkage is possible if θ is small enough and if δ is large enough, because the lost payoff from the risky front is more important the smaller is θ and the larger is δ.

The reason that collusion sustained through linkage always is profitable in this case and not in the case in Proposition 5 is because sustaining collusion on the safe front by linking it with the risky front does not put any collusive payoff on the risky front at additional risk of being lost. This is true because there is never going to be a real or apparent deviation on the safe front if collusion is sustainable. In contrast, in Proposition 5 linking the risky and safe fronts could be unprofitable because noise on the risky front put the collusive payoff on the safe front at risk.

4 Conclusion

This paper examines the effect on tacit collusion of the interaction between imperfect information and multimarket contact. We consider the highest sustainable payoff for two players engaged in repeated play on two fronts, one that is subject to unobserved payoff-relevant shocks and one that is not. We completely characterize the ability to sustain collusion through strategic linkage of the two fronts. While in some instances collusion that is sustained through linkage is more profitable than when linkage is not possible, we find in other instances that collusion sustained through linkage actually leads to lower payoffs. This surprising result occurs because destabilizing shocks are contagious, in the sense that the strategic linkage allows the shocks' adverse effects to spread from one front to another. We show these results using simple grim strategies, then illustrate their robustness by using optimal strategies that yield the maximal payoffs associated with symmetric sequential equilibria. Importantly, every instance in which strategic linkage is harmful is one in which the intuition from perfect information models predicts that it helps. To be precise, linkage does help the players to sustain collusion on both fronts in the settings we identify, but the cost of the punishment necessary to support that collusion outweighs the benefit.

Our finding that exploiting multimarket contact by strategically linking markets may be unprofitable is useful to parties involved in antitrust enforcement and litigation, who might otherwise argue that the creation of cross-market linkages necessarily increases firms' market power. It also is useful to firms considering either strategic linkage of existing markets or expansion into new markets with the hope of sustaining higher degrees of collusion. While the scope of collusion can be expanded through linkage, the cost of doing so may be too great. Finally, our results provide insights to governments trying to find the correct interaction between issue areas such as various trade and military policies, who might otherwise let small conflicts cascade into larger ones. Of particular importance is the danger posed by even arbitrarily small linkages, of the sort associated with the practice of brinkmanship.

Appendix

Lemma 1 provides the essential analysis for Propositions 3 through 6, which concern the effect of multimarket contact when the players use optimal strategies. It also allows the use of a public randomization device in the first period of collusive play.

Lemma 1 *Assume that the players use optimal linked strategies. If*

$$\delta\left(x_s - z_s\right) + (1-\theta)\,\delta\left(x_r - z_r\right) < \left(x_s - w_s\right) + \left(x_r - w_r\right),$$

then collusion on both fronts cannot be sustained through linkage, and

$$\overline{V}^o_{rs} = \overline{V}^o_r + \overline{V}^o_s.$$

If

$$\delta\left(x_s - z_s\right) + (1-\theta)\,\delta\left(x_r - z_r\right) \geq \left(x_s - w_s\right) + \left(x_r - w_r\right),$$

then collusion on both fronts can be sustained through linkage. In this case, if the collusive SSE begins with both players selecting C on each front with probability 1, then the maximal collusive payoff sustained through linkage is

$$\overline{V}^o_{rs} = \frac{w_r - \theta\left(x_r - z_r\right)}{1-\delta} + \frac{w_s}{1-\delta},$$

with

$$V^p_r = \frac{(1-\theta)\delta x_r + \theta\delta z_r - (x_r - w_r)}{\delta(1-\delta)} + \frac{w_s}{1-\delta}.$$

However, if $w_r - \theta\left(x_r - z_r\right) < z_r$, then the players' payoffs will be higher by abandoning any attempt to collude on the risky front, no matter how infrequently.

Proof of Lemma 1: Suppose that the players employ linked strategies. Let $\mathcal{V}_{rs} \in \Re$ denote the non-empty and compact set of all SSE payoffs for the coordinated game, where the subscript "rs" denotes the linkage of the risky and safe fronts, and let $\overline{\sigma}_{rs}$ denote the SSE that yields the largest element of $\mathcal{V}_{rs}$, $\overline{V}^o_{rs}$. In the first period of play, $\overline{\sigma}_{rs}$ specifies that with probability α_{ij} the firms select action i on the risky front and action j on the safe front, where $i, j \in \{C, D\}$ and $\sum \alpha_{ij} = 1$. As has been previously observed, when the strategy specifies selecting C on the risky front, in equilibrium there are only two common knowledge events following the choice of actions in a period: either both players received at least w_r on the risky front, or at least one player did not. $\overline{\sigma}_{rs}$ specifies two SSEs truncated to the remaining game, one for each common knowledge contingency. These two SSEs are denoted σ^c and σ^p_r, with associated payoffs V^c and V^p_r. The SSEs specified by $\overline{\sigma}_{rs}$ following a defection on the safe front or on both the risky and safe fronts are denoted σ^p_s and σ^p_{rs}. These SSEs come into play only out of equilibrium, and they have associated payoffs V^p_s and V^p_{rs}. The payoff to $\overline{\sigma}_{rs}$ is

$$\begin{aligned} \overline{V}^o_{rs} = {} & \alpha_{CC} \{(1-\theta)[w_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^p_r]\} + \\ & \alpha_{CD} \{(1-\theta)[w_r + z_s + \delta V^c] + \theta [z_r + z_s + \delta V^p_r]\} + \\ & \alpha_{DC} \{(1-\theta)[z_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^c]\} + \\ & \alpha_{DD} \{(1-\theta)[z_r + z_s + \delta V^c] + \theta [z_r + z_s + \delta V^c]\}. \end{aligned} \tag{28}$$

We wish to maximize $\overline{V}^o_{rs}$ with respect to V^p_r, V^p_s, V^p_{rs}, V^c, and the randomization probabilities, subject to the following incentive constraints.

$$(1-\theta)[w_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^p_r] \geq (1-\theta)[x_r + w_s + \delta V^p_r] + \theta [z_r + w_s + \delta V^p_r] \tag{29}$$

$$(1-\theta)[w_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^p_r] \geq (1-\theta)[w_r + x_s + \delta V^p_s] + \theta [z_r + x_s + \delta V^p_{rs}] \tag{30}$$

$$(1-\theta)[w_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^p_r] \geq (1-\theta)[x_r + x_s + \delta V^p_{rs}] + \theta [z_r + x_s + \delta V^p_{rs}] \tag{31}$$

$$(1-\theta)[w_r + z_s + \delta V^c] + \theta [z_r + z_s + \delta V^p_r] \geq (1-\theta)[x_r + z_s + \delta V^p_r] + \theta [z_r + z_s + \delta V^p_r] \tag{32}$$

$$(1-\theta)[z_r + w_s + \delta V^c] + \theta [z_r + w_s + \delta V^c] \geq (1-\theta)[z_r + x_s + \delta V^p_s] + \theta [z_r + x_s + \delta V^p_s] \tag{33}$$

The incentive constraints (29)-(31) require that, when C is specified on both fronts, it must not be profitable for a player to deviate on the risky front only, the safe front only, or both the risky and safe fronts, respectively. Constraint (32) requires that, when C is specified only on the risky front, it must not profitable for a player to deviate on the risky front only. Similarly, constraint (33) requires that, when C is specified only on the safe front, it must not be profitable for a player to deviate on the safe front only.

By familiar reasoning, we can show that $V^c = \overline{V}^o_{rs}$. Moreover, because decreasing either V^p_s or V^p_{rs} does not affect the objective function but relaxes the constraints, it must be the case that $V^p_s = V^p_{rs} = \frac{z_r+z_s}{1-\delta}$, the lowest possible payoff satisfying the individual rationality constraint. Using these two preliminary results, the problem may be simplified to maximize

$$\overline{V}^o_{rs} = \frac{[\alpha_{CC}+\alpha_{DC}]w_s+[\alpha_{CC}+\alpha_{CD}][(1-\theta)w_r+\theta z_r]+[\alpha_{CD}+\alpha_{DD}]z_s+[\alpha_{DC}+\alpha_{DD}]z_r+[\alpha_{CC}+\alpha_{CD}]\theta\delta V^p_r}{1-(1-[\alpha_{CC}+\alpha_{CD}]\theta)\delta}, \tag{34}$$

subject to the following constraints.

$$\begin{aligned}
\delta\left[\overline{V}^o_{rs} - V^p_r\right] &\geq x_r - w_r && (35)\\
(1-\theta)\delta\left[\overline{V}^o_{rs} - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V^p_r - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s && (36)\\
(1-\theta)\delta\left[\overline{V}^o_{rs} - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V^p_r - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s + (1-\theta)(x_r - w_r) && (37)\\
\delta\left[\overline{V}^o_{rs} - V^p_r\right] &\geq x_r - w_r && (38)\\
\delta\left[\overline{V}^o_{rs} - \frac{z_r+z_s}{1-\delta}\right] &\geq x_s - w_s && (39)
\end{aligned}$$

Constraints (35)-(39) are updated versions of constraints (29)-(33). It is clear that (38) is superfluous. That is, if defection on the risky front only is deterred when collusion is specified on both the safe and risky fronts, then defection on the risky front is deterred when collusion is specified only on the risky front. Also, if (37) holds, then both (36) and (39) hold. That is, if defection on both fronts is deterred when collusion is specified on both fronts, then defection on the safe front only is deterred when collusion is specified either on both fronts or only on the safe front. Therefore, the only relevant incentive constraints are (35) and (37). Finally, (35) must bind. If it did not, then one could increase V^p_r, relax (37), still satisfy (35), yet increase $\overline{V}^o_{rs}$. That is, the punishment for an apparent deviation on the risky front only should not be more severe than is necessary to prevent such deviations. With these intermediate results, the maximal collusive payoff with linkage of the safe and risky fronts is found by maximizing

$$\overline{V}^o_{rs} = \frac{[\alpha_{CC}+\alpha_{DC}]w_s+[\alpha_{CC}+\alpha_{CD}][(1-\theta)w_r+\theta z_r]+[\alpha_{CD}+\alpha_{DD}]z_s+[\alpha_{DC}+\alpha_{DD}]z_r+[\alpha_{CC}+\alpha_{CD}]\theta\delta V^p_r}{1-(1-[\alpha_{CC}+\alpha_{CD}]\theta)\delta} \tag{40}$$

with respect to α_{CC}, α_{CD}, α_{DC}, α_{DD}, and V_r^p, subject to the following incentive constraints.

$$\delta\left[\overline{V}_{rs}^o - V_r^p\right] = x_r - w_r \tag{41}$$

$$(1-\theta)\delta\left[\overline{V}_{rs}^o - \frac{z_r+z_s}{1-\delta}\right] + \theta\delta\left[V_r^p - \frac{z_r+z_s}{1-\delta}\right] \geq x_s - w_s + (1-\theta)(x_r - w_r) \tag{42}$$

Moreover, individual rationality requires that $V_r^p \geq \frac{z_r+z_s}{1-\delta}$.

Using δV_r^p from (41), we have

$$\overline{V}_{rs}^o = \frac{[\alpha_{CC}+\alpha_{CD}]\,[w_r - \theta(x_r - z_r)] + [\alpha_{DC}+\alpha_{DD}]\,z_r + [\alpha_{CC}+\alpha_{DC}]\,w_s + [\alpha_{CD}+\alpha_{DD}]\,z_s}{1-\delta}$$

and

$$\delta\left[\overline{V}_{rs}^o - \frac{z_r+z_s}{1-\delta}\right] \geq (x_s - w_s) + (x_r - w_r)\,. \tag{43}$$

That the individual rationality constraint $V_r^p \geq \frac{z_r+z_s}{1-\delta}$ is satisfied is shown by substituting V_r^p from (41) into (43).

Finally, $\overline{V}_{rs}^o$ can be further simplified by recalling that $\alpha_{DD} = 1 - \alpha_{CC} - \alpha_{CD} - \alpha_{DC}$ to yield

$$\overline{V}_{rs}^o = \frac{z_r + z_s + [\alpha_{CC}+\alpha_{CD}]\,[w_r - \theta(x_r - z_r) - z_r] + [\alpha_{CC}+\alpha_{DC}]\,[w_s - z_s]}{1-\delta}.$$

Taking the derivative of the preceding expression with respect to α_{CC}, α_{CD}, and α_{DC} yields the following.

$$\frac{d\overline{V}_{rs}^o}{d\alpha_{CC}} = \frac{[w_r - \theta(x_r - z_r) - z_r] + [w_s - z_s]}{1-\delta}$$

$$\frac{d\overline{V}_{rs}^o}{d\alpha_{CD}} = \frac{[w_r - \theta(x_r - z_r) - z_r]}{1-\delta}$$

$$\frac{d\overline{V}_{rs}^o}{d\alpha_{DC}} = \frac{[w_s - z_s]}{1-\delta}$$

In terms of payoff maximization, it is clear that one should choose $\alpha_{CC} = 1$ if $w_r - z_r > \theta(x_r - z_r)$ and $\alpha_{DC} = 1$ otherwise.

If $(1-\theta)\,\delta(x_r - z_r) \geq (x_r - w_r)$, then one can show that $w_r - z_r > \theta(x_r - z_r)$. Therefore, if collusion is possible on the risky front using unlinked strategies, then the players always should attempt to collude on both fronts. The only time at which attempting collusion on both fronts might be unprofitable is when collusion is not possible on the risky front using unlinked strategies. What the analysis of the randomization probabilities reveals is that if $w_r - z_r < \theta(x_r - z_r)$, then attempting to collude on the risky front with even arbitrarily low frequency is less profitable than is abandoning collusion there entirely.

Ignoring the preceding analysis for a moment, we also are interested in the maximum payoff when the players are assumed to begin collusion by selecting C on each front with probability 1. Therefore, we select $\alpha_{CC} = 1$, which yields

$$\overline{V}^o_{rs} = \frac{w_r - \theta(x_r - z_r) + w_s}{1 - \delta},$$

provided that (43) is satisfied. Simple substitution reveals that (43) is satisfied when

$$\delta(x_s - z_s) + (1 - \theta)\delta(x_r - z_r) \geq (x_s - w_s) + (x_r - w_r),$$

which is the desired result.

Proof of Proposition 3: If collusion cannot be sustained on either front in the unlinked equilibrium, then $(1 - \theta)\delta(x_r - z_r) < x_r - w_r$ and $\delta(x_s - z_s) < x_s - w_s$. Therefore,

$$\delta(x_s - z_s) + (1 - \theta)\delta(x_r - z_r) < (x_s - w_s) + (x_r - w_r).$$

From Lemma 1 we conclude that collusion cannot be sustained through linkage and that $\overline{V}^o_{rs} = \frac{z_r + z_s}{1-\delta} = \overline{V}^o_r + \overline{V}^o_s$.

Proof of Proposition 4: If collusion can be sustained on both fronts in the unlinked equilibrium, then $(1 - \theta)\delta(x_r - z_r) \geq x_r - w_r$ and $\delta(x_s - z_s) \geq x_s - w_s$. Therefore,

$$\delta(x_s - z_s) + (1 - \theta)\delta(x_r - z_r) \geq (x_s - w_s) + (x_r - w_r).$$

From Lemma 1 we conclude that collusion can be sustained through linkage and that

$$\overline{V}^o_{rs} = \frac{w_r - \theta(x_r - z_r) + w_s}{1 - \delta} = \overline{V}^o_r + \overline{V}^o_s.$$

Proof of Proposition 5: If collusion can be sustained on the safe front but not on the risky front in the unlinked equilibrium, then $\delta(x_s - z_s) \geq x_s - w_s$ and $(1 - \theta)\delta(x_r - z_r) < x_r - w_r$. If

$$\delta(x_s - z_s) + (1 - \theta)\delta(x_r - z_r) < (x_s - w_s) + (x_r - w_r),$$

then from Lemma 1 we conclude that collusion cannot be sustained through linkage. If

$$\delta\left(x_s - z_s\right) + \left(1-\theta\right)\delta\left(x_r - z_r\right) \geq \left(x_s - w_s\right) + \left(x_r - w_r\right),$$

then from Lemma 1 we conclude that collusion can be sustained through linkage and that

$$\overline{V}^o_{rs} = \frac{w_r - \theta\left(x_r - z_r\right) + w_s}{1-\delta}.$$

If $\theta\left(x_r - z_r\right) > w_r - z_r$, then $\overline{V}^o_{rs} < \overline{V}^o_r + \overline{V}^o_s$, which implies that collusion sustained through linkage strictly reduces the players' payoffs.

Proof of Proposition 6: If collusion can be sustained on the risky front but not on the safe front in the unlinked equilibrium, then $\left(1-\theta\right)\delta\left(x_r - z_r\right) \geq x_r - w_r$ and $\delta\left(x_s - z_s\right) < x_s - w_s$. If

$$\delta\left(x_s - z_s\right) + \left(1-\theta\right)\delta\left(x_r - z_r\right) < \left(x_s - w_s\right) + \left(x_r - w_r\right),$$

then from Lemma 1 we conclude that collusion cannot be sustained through linkage. If

$$\delta\left(x_s - z_s\right) + \left(1-\theta\right)\delta\left(x_r - z_r\right) \geq \left(x_s - w_s\right) + \left(x_r - w_r\right),$$

then from Lemma 1 we conclude that collusion can be sustained through linkage and that

$$\overline{V}^o_{rs} = \frac{w_r - \theta\left(x_r - z_r\right) + w_s}{1-\delta} > \frac{w_r - \theta\left(x_r - z_r\right)}{1-\delta} + \frac{z_s}{1-\delta} = \overline{V}^o_r + \overline{V}^o_s.$$

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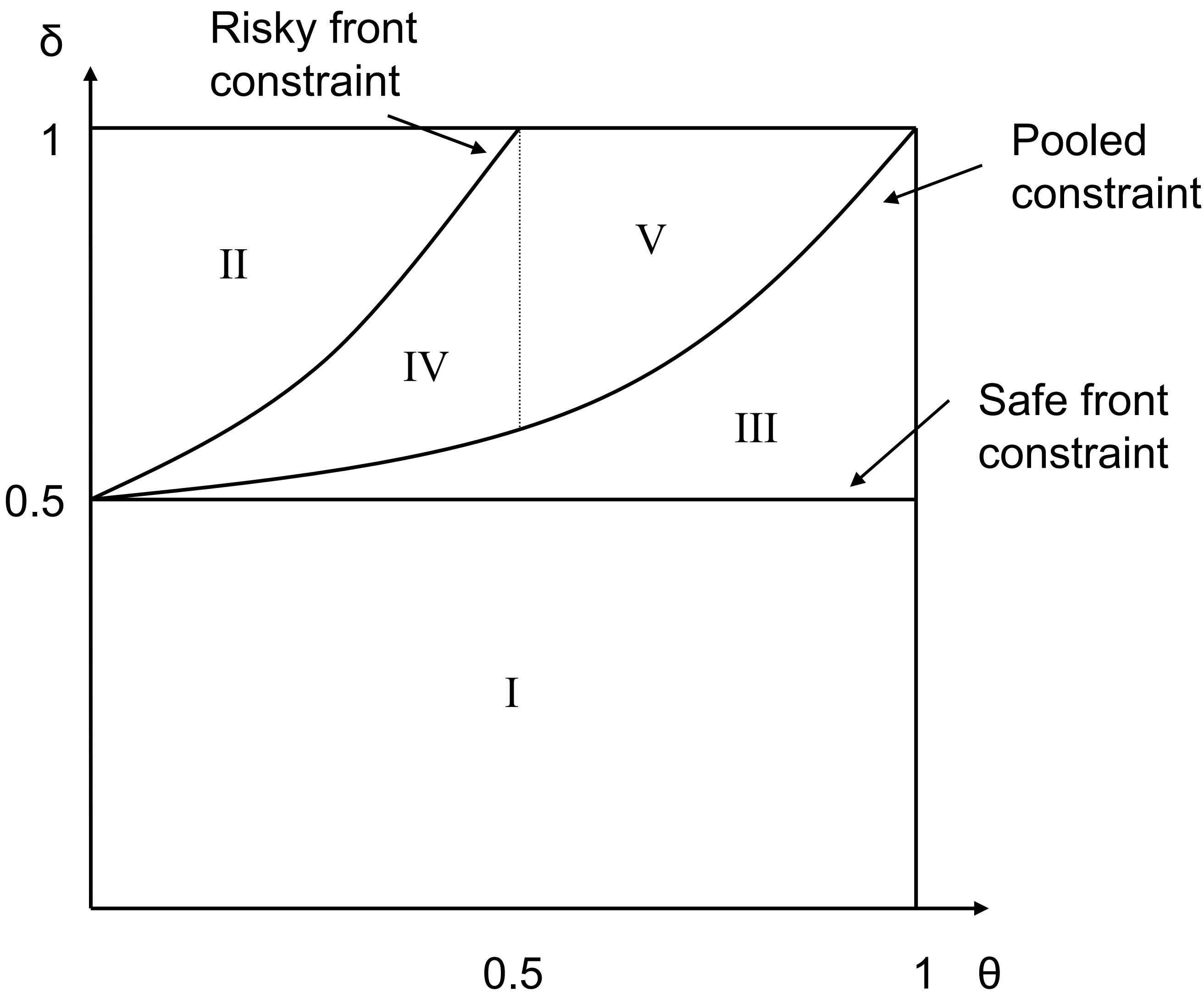

Figure 1: Sustainability of Collusion with Optimal Punishments in Different Regions of (θ, δ) Space.
Consider the following payoffs: $w_r = w_s = 30$; $x_r = x_s = 50$; $y_s = 0$; $z_r = z_s = 10$. Analysis of linked and unlinked equilibria with optimal strategies yields the following regions of (θ, δ) space.
Region I: Collusion not sustainable on either front in both unlinked and linked equilibria.
Region II: Collusion sustainable on both fronts in both unlinked and linked equilibria.
Region III: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front not sustainable through linkage.
Region IV: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front sustainable through linkage, and profitable.
Region V: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front sustainable through linkage, but unprofitable.

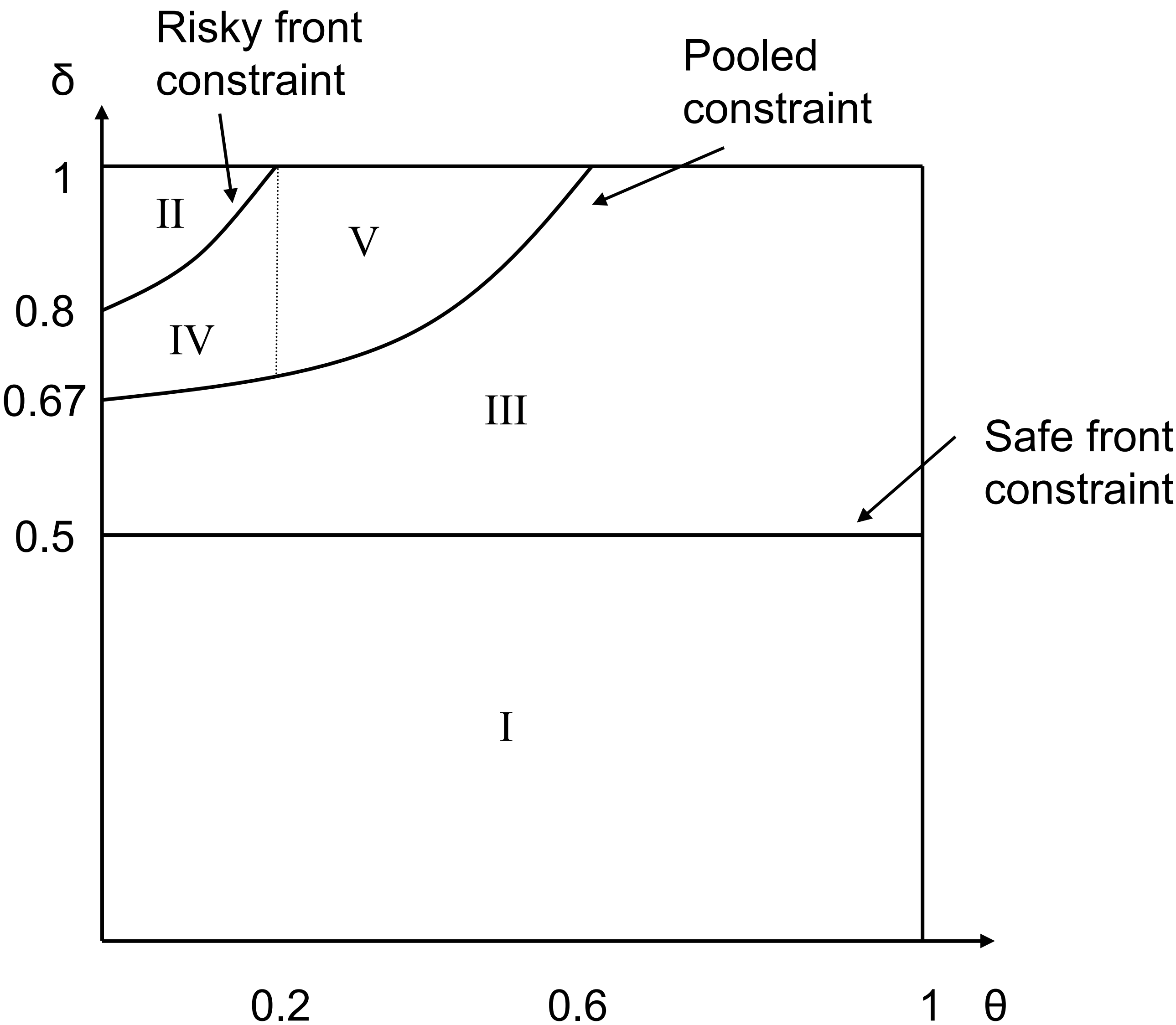

Figure 2: Sustainability of Collusion with Optimal Punishments in Different Regions of (θ, δ) Space.
Consider the following payoffs: $w_r = 20$; $w_s = 30$; $x_r = 60$; $x_s = 50$; $y_s = 0$; $z_r = z_s = 10$. Analysis of linked and unlinked equilibria with optimal strategies yields the following regions of (θ, δ) space.
Region I: Collusion not sustainable on either front in both unlinked and linked equilibria.
Region II: Collusion sustainable on both fronts in both unlinked and linked equilibria.
Region III: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front not sustainable through linkage.
Region IV: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front sustainable through linkage, and profitable.
Region V: Collusion sustainable on safe front but not on risky front in unlinked equilibrium. Additional collusion on risky front sustainable through linkage, but unprofitable.