Internal cartel stability with time-dependent detection probabilities.

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Abstract

To account for the illegal nature of price-fixing agreements per-period detection probabilities that can vary over time are introduced in a dynamic oligopoly. The resulting ICCs for internal cartel stability indicate that for discount factors up to 10% per-period detection probabilities of 5% are needed to reduce the number of cartel members by 50%. For the special case of stationary supergames with constant per-period detection probabilities p elegant rules emerge: internal cartel stability requires the discount factor to increase with $100 \times p/(1-p)$ percent while a fixed fine of $100 \times (1-p)/p$ percent of incremental cartel profits is required for making the ICC always binding.

Key words: Internal cartel stability, trigger strategy, detection probabilities, non-stationary supergames.

JEL Classification: L12, L41

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1 Introduction

Ever since the writings of Adam Smith economists are aware of firms' desire for coordinating their actions towards higher profits, coordination that typically is at the expense of consumer welfare. In his often-quoted intuition as to these coordinated actions Smith talks of "conspire",[1] which captures nicely the fact that these coordinated actions are illegal although at the time of Smith's writing there was no such thing as antitrust law.

Before conspiring firms can be expected to assess to pros and cons of joining such a cartel. An important setback is the possibility that the agreement is not adhered to by all partners; if every cartel member quotes the collusive price anyone member has an incentive to quote a lower price in order to capture a larger market share than its assigned quota, and to reap large short-run profits. As collusion is illegal no enforceable contract can be written that prevents this type of defection.

With the introduction of his "trigger strategy", Friedman (1971) specified the conditions under which defection is less profitable than complying to the cartel agreement. In particular, if future period payoffs are valued 'enough' the short-term gains from defection are outweighed by the long-term gains from compliance, inducing the cartel to be stable internally. Meanwhile applications of Friedman-type trigger strategies abound, including the analysis of price collusion and the concomitant choice of location (Friedman and Thisse, 1993), collusion by multi-market oligopolists in one market to dampen procompetitive regulation in another market (Phillips and Mason, 1996), collusion under different degrees of product differentiation (Ross, 1992), collusion in sealed bid second price auctions (von Ungern-Sternberg, 1988), and collusion when competition is local (Verboven, 1998).[2]

Antitrust laws that are actively enforced imply as such another setback for cartels: the possibility of being detected. Indeed, based on a sample some

[1]Smith (1776, 1986, p. 232):

> People of the same trade seldom meet together, even for meritment and diversion, but the conversation ends in a long conspiracy against the public, or in some contrivance to raise prices...

[2]An often-stated criticism is that Friedman's trigger strategy implies a too harsh punishment scheme for it to be realistic. In response to this critique a number of trigger strategies is developed with more intense but shorter lived punishment regimes, such as the "stick-and-carrot strategy" of Abreu (1986) and the "repentance strategy" of Segerstrom (1988). These strategies are applied much less however, in part because of their assumed hyperrational behaviour of individual agents. Also, experimental research shows that subjects involved in repeated prisoner's-dilemma-type stage games respond to defection with non-cooperative Nash behaviour for ever after (Mason and Phillips, 2002).

184 price fixing cartels that were indicted by the US Department of Justice from 1961 through 1988, Bryant and Eckard (1991) estimate the maximum probability of getting caught in any given year at 13% to 17%.

Few papers have analyzed the impact of detection probabilities on internal cartel stability. These include Motta and Polo (2003), Rey (2003), and Spagnolo (2004). A unifying theme of these studies is their examination of the impact of corporate leniency programs on internal cartel stability.[3] For that an exogenous, constant, per-period cartel detection probability is introduced in a supergame of collusion. One finding within this literature is that leniency programs can be pro-collusive as they reduce the downside of detection. A related contribution is Martin (2004) who explicitly considers the cost of maintaining a collusive agreement (private enforcement cost). With uncertain demand and an antitrust authority that starts investigating a potential cartel whenever price exceeds some threshold he finds that a trigger strategy sustains a coordinated price below the monopoly price only in order to make defection less profitable.

Quite a different approach is taken in Harrington (2003, 2004a, 2004b) and Harrington and Chen (2004). There the focus is on pricing dynamics of cartels whereby members take into account that the probability of detection increases with the amplitude of price changes over time. This has the general effect of price paths becoming more smooth as both upward price adjustments (towards some collusive price) and downward price changes (towards some punishment price) are kept within boundaries to avoid suspicion.

In this paper internal cartel stability is also analyzed under the presence of an active antitrust authority. For that a mechanism is introduced that generates for every period a probability that the cartel is discovered. Per-period detection probabilities are thus allowed to vary over time thereby yielding a non-stationary supergame. Indeed, in practise it is not expected that per-period detection probabilities are time invariant as they are typically influenced by time-varying factors, including the number of firms forming the cartel, the level of transparency in the market, and the nature and abundance of resources available to antitrust authorities.

The set-up obviously yields a generalization of the ICC for internal cartel stability. In particular, both the discount rate and all per-period detection probabilities have to be 'low enough' for a cartel to be stable internally. Numerical simulations indicate that for discount factors up to 10% per-period detection probabilities of little over 5% are needed to reduce the number of firms that can sustain a cartel by 50% or more. Also, it is shown that an

[3]Leniency programs provide for fine reductions (up to 100%) to cartel members that report the cartel to the antitrust authorities (see also Hinloopen, 2003).

increase *in any* future per-period detection probability reduces the domain for which the strictest ICC for internal cartel stability is not binding.

Developments such as the evolution of jurisprudence or the increased effectiveness with which antitrust authorities prosecute cartels could lead to weakly increasing per-period detection probabilities. This particular situation is shown to coincide with the stationary version of the supergame whereby per-period detection probabilities are constant over time. For this case an elegant rule emerges: given a constant per-period detection probability $p \in (0, 1)$ the minimum discount factor for internal cartel stability increases with $100 \times p/(1-p)$ percent. This rule does not depend on any specification of firm profits other than some qualitative ordering. For example, if detection in any given year occurs with a 17% probability the minimum discount factor for internal cartel stability increases by 20%.

Additional results are obtained when fixed fine payments are introduced. Prospective fine payments and detection probabilities appear to be substitutable instruments as an increase in either reduces the domain for which the strictest ICC for internal cartel stability is not binding. At the same time the two instruments are complementary in that an increase in prospective fine payments yields more effect the higher are per-period detection probabilities, while an increase in any per-period detection probability yields more effect the higher are prospective fine payments.

Also now a simple rule of thumb arises for the stationary example which does not depend on functional forms: given a constant per-period detection probability $p \in (0, 1)$ a fixed fine of $100 \times (1-p)/p$ percent of incremental cartel profits makes the ICC for internal cartel stability always binding. Using again the example of the 17% per-period detection probability indicates that fines of about five times incremental cartel profits are required for them to break cartels.

In the next section the model is specified, expected cartel adherence payoffs are derived as well as those that come with defection in any future period. The concomitant ICCs for internal cartel stability are the subject of Section 3. Changing per-period detection probabilities are considered next and in Section 5 fixed fines are introduced. Section 6 concludes.

2 Set up

At some point in time, i.e. $t = 1$, a group of $m \geq 2$ symmetric firms start colluding by quoting a joint price. The internal stability of this agreement hinges exclusively on each individual cartel member's assessment of expected payoffs that come with quoting the agreed upon price (compliance) and those that come with quoting any other possible price (defection).

2.1 Detection probabilities

As of the moment that the cartel is formed each individual cartel member knows the probability that the antitrust authorities discover it in the current and any future period.[4] For period t this detection probability equals $p_t \in (0,1)$, for $t = 1, 2, ...$, which is drawn from some discrete probability density function, $f(x; \boldsymbol{\theta}(t))$ for $x = \{\textit{detection, no detection}\}$. As alluded to above Bryant and Eckard (1991) estimate p_t to be at most between 0.13 and 0.17 for any given year. The parameter vector $\boldsymbol{\theta}(t)$ contains all information that could affect the distribution of the per-period detection probabilities, such as the number of firms that has joined the cartel, the nature and abundance of resources available to the antitrust authorities, and the level of transparency in the market where the cartel operates.[5] Through $\boldsymbol{\theta}(t)$ the per-period detection probabilities are possibly also influenced by the number of periods that the cartel exists.[6]

Given the per-period detection probabilities, the probability that the cartel is discovered as of period t through period $t+k$ is given by (see also Figure 1):

$$P_t(k) = 1 - \prod_{j=0}^{k}(1 - p_{t+j}). \tag{1}$$

Note that $P_t(0) = p_t$, and that $P_t(k)$ is increasing in k, with $\lim_{k\to\infty} P_t(k) = 1$; looking ahead 'far enough' always yields detection to be a certain event provided that there is a strictly positive per-period detection probability during each period. Yet, because future cash flows (and fine payments, see Section 5 below) are discounted, (1) does not imply that defection is always preferred over compliance.

[4]Throughout we assume that detection implies not only that the cartel is discovered, but also that the antitrust authorities were able to provide all necessary legal proof for it to be dismantled.

[5]Within a context of colluding firms gradually adjusting their price towards the target level, $\boldsymbol{\theta}(t)$ would also be affected by the size of consecutive price adjustments (see Harrington (2003, 2004a, 2004b) and Harrington and Chen, 2004).

[6]In Section 4.1 the special case of weakly increasing per-period detection probabilities is analyzed in detail.

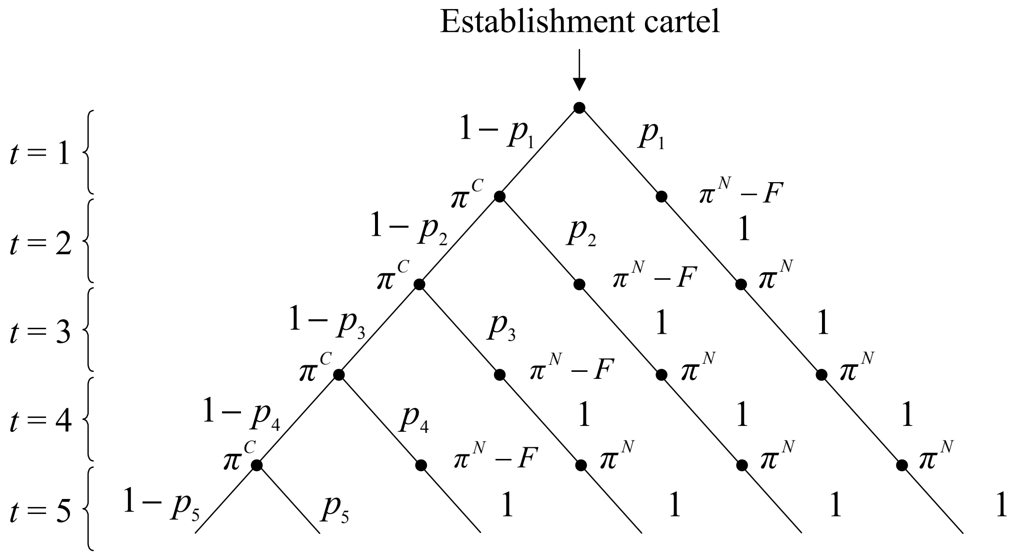

Figure 1: Development over time of expected cartel-adherence payoff.

2.2 Compliance: strategy and expected payoff

Let p^N be the noncooperative Nash equilibrium price with an associated vector of quantities, $(q_1^N, ...q_n^N)$ such that $q_i^N \geq 0$, and $\sum_{i=1}^n q_i^N = D(p^N)$.[7] This price can be anything as long as it is the result of individual profit-maximizing behaviour only. Likewise, let $p^c \in (p^N, p^m]$ be the agreed upon collusive price, where $p^m > p^N$ is the monopoly price. For supporting this coordinated equilibrium as a subgame perfect Nash equilibrium (SPNE) recall Friedman's (1971) trigger strategy profile:

$$\begin{aligned} s_i^1 &= p^c, \\ s_i^t &= \begin{cases} p^c \; if \; p_j^k = p^c, \; k = 1, ..., t-1, \; j = 1, ..., m, \\ p^N \; otherwise; \end{cases} \end{aligned} \tag{2}$$

$t = 2, ...,\ i = 1, ..., m$. The difference here with Friedman (1971) is that in addition to defection, during each period the cartel can be discovered by the antitrust authorities. If this happens all cartel members realize noncooperative Nash equilibrium profits of the stage game only and the collusive agreement is dismantled for good. Hence, retaliation to the noncooperative

[7] In case actions are identical across firms subscripts are ignored.

Nash equilibrium occurs both after defection and when the cartel is discovered. In the latter case it is, of course, not so much retaliation but an equilibrium forced upon the industry by the antitrust authorities.[8]

Per-firm single-period noncooperative Nash equilibrium profits are denoted by $\pi^N = \left[D(p^N) - AC(q(p^N))\right] q(p^N)$; per-firm single-period collusive profits are denoted by $\pi^c = \left[D(p^c) - AC(q(p^c))\right] q(p^c)$. For engaging in coordinated actions to be profitable let $0 \leq \pi^N < \pi^c$. Cartel compliance profits earned during period $t + k$ as expected at the beginning of period t then equal:

$$v_t^c(k) = \left[1 - P_t(k)\right] \pi^c + P_t(k)\pi^N, \tag{3}$$

where $\lim_{k\to\infty} v_t^c(k) = \pi^N$. Since $P_t(k)$ is increasing in k it is immediate that the expected per-period value of cartel compliance is decreasing in k; the more distant is the future period considered, the lower are expected per-period compliance profits (abstaining from any discounting).

If future periods are discounted with rate $\delta \in [0, 1] \subset \mathbb{R}$, the expected present discounted value at the beginning of period t of cartel compliance is given by:

$$V_t^c = \sum_{i=0}^{\infty} \delta^i v_t^c(i). \tag{4}$$

2.3 Defection

Whether or not the above defined trigger strategy can sustain the coordinated equilibrium as a SPNE depends on the alternative strategies that are open to individual firms. With probability $(1 - p_{t+d})$ firm i earns defection profits $\pi_i^{defect} = \left[D(p_i^{defect}, p_{-i}^c) - AC(q(p_i^{defect}, p_{-i}^c)\right] q(p_i^{defect}, p_{-i}^c)$ if it defects at the beginning of period $t + d$. To make defection credible let $\pi^{defect} > \pi^c$. During the same period the antitrust authorities can still find the defecting firm guilty of having participated in the illegal cartel, a finding that occurs with probability p_{t+d}.[9] If this happens the defecting firm (and all other cartel

[8] This follows Harrington (2003, 2004a, 2004b) and Harrington and Chen (2004). Assuming that in the period during which the cartel is discovered firms earn non-cooperative Nash profits rather than collusive profits is for computational convenience only; it does not affect any of the results reported below.

[9] In fact, retaliation typically involves relatively large reductions in price and could therefore attract the attention of the antitrust authorities (see Abrantes-Metz *et al.*, 2004). We ignore however the possible effect of defection on the probability that the cartel is discovered during the defection period (see Hinloopen (2004) for an explicit treatment of this issue; see also footnote 5).

members) earns π^N.[10] Following Friedman (1971) assume that defection is always observed, also if in the same period the cartel is not discovered by the antitrust authorities. Accordingly, defection by one cartel member is always followed by reversion to non-cooperative Nash behaviour of all cartel members as of the next period for ever after.

Defection profits earned during period $t+d$ as expected at the beginning of period t then equal:

$$v_t^{defect}(d) = [1 - P_t(d)]\,\pi^{defect} + P_t(d)\pi^N, \tag{5}$$

where $\lim_{d\to\infty} v_t^{defect}(d) = \pi^N$, and $\partial v_t^{defect}(d)/\partial d < 0$; the more distant is the period during which defection is envisaged, the less likely it is that defection is still an option, the lower is the expected payoff for that period.

The expected present value at the beginning of period t of defection during period $t+d$ then equals:

$$V_t^{defect}(d) = \sum_{i=0}^{d-1} \delta^i v_t^c(i) + \delta^d v_t^{defect}(d) + \sum_{i=d+1}^{\infty} \delta^i \pi^N. \tag{6}$$

3 Internal cartel stability

Comparing (4) with (6) yields the ICCs for (2) to sustain collusion as a SPNE (see also Appendix 1):

$$\frac{\pi^{defect} - \pi^c}{\pi^c - \pi^N} \leq \sum_{i=1}^{\infty} \delta^i \prod\nolimits_{j=1}^{i} (1 - p_{t+d+j}) = S(t+d), \tag{7}$$

$t = 1, 2, ..., d = 1, 2, ...$ Obviously, $S(t+d) = S(t'+d')$ for any t' and d' such that $t'+d' = t+d$. Accordingly, denote the RHS of (7) by $S(k)$, with $k = t+d$. It is useful to state two properties of $S(k)$:

Lemma 1 *If $\delta \in [0,1] \subset \mathbb{R}$ and $\exists\ t > 1 \mid p_t \in (0,1) \subset \mathbb{R}$, then $\forall\ k > 1$, (i) $S(k) \neq \emptyset$, and (ii) $S(k)$ is a monotone mapping from $\mathbb{N}_+$ onto $\mathbb{R}_+$.*

Proof. See Appendix 2 ∎

Lemma 1 implies that the minimum over k of $S(k)$ exists and that it is unique. For future reference, let $S(k^*) = \min_{\{k\}} S(k)$, $k > 1$. Accordingly, if (7) is not binding for k^* the cartel will not break down due to the probability

[10] Assuming that the defecting firm earns π^{defect} with probability $(1 - p_{t+d})$ only is for computational convenience; assuming that the defecting firm earns π^{defect} with certainty would not affect any of our conclusions stated below.

that it is discovered by the antitrust authorities. Likewise, the existence of an active antitrust authority as such does not necessarily preclude cartels being formed in the first place.

On the other hand, the folk theorem on infinitely repeated stage games being able to sustain any (coordinated) equilibrium as a SPNE provided that discount rates for future revenues are 'low enough', is qualified; (7) could be violated for k^* for low values of δ if at least one per-period detection probability is strictly positive and 'large enough'.

As the ICCs (7) relate to a non-stationary supergame Table 1 contains some illustrative numerical simulations.[11] It displays the maximum number of symmetric firms for which the cartel is internally stable for different discount rates and different ranges of the per-period detection probabilities. Cartel members are assumed to produce a homogeneous good, all using the same technology, and competition is over price. The LHS of (7) then equals $m - 1$.[12]

These numerical simulations are executed as follows. First, a sequence of 10^4 per-period detection probabilities is generated, whereby $f(x; \boldsymbol{\theta}(t)) \sim N(\mu, \sigma)$ and constructed such that all probabilities are within the pre-specified range. Next, given a discount factor δ the value of $S(k)$ is calculated for $k = \{1, 2, ..., 10^4\}$ and $i = 10^3$, thus yielding $S(k^*)$.[13] This process is then replicated 10^3 times and the mean of the resulting 10^3 different values for $S(k^*)$ is displayed in Table 1.[14]

The results in Table 1 show that small per-period detection probabilities already have quite a big impact on the internal stability of cartels. For discount factors up to 10% per-period detection probabilities of little over 5% are needed to reduce the number of firms that can sustain a cartel by 50% or more. If the per-period detection probability is in the order of 15% (see Bryant and Eckard, 1991), on average a cartel will be stable internally for some 4 to 6 firms.

[11] GAUSS routines with which these simulations are carried out are available upon request.

[12] Note that in this case $\pi^N = 0$, and that defection implies a price p_i^{defect} just below the collusive price such that the defecting firm captures the entire market.

[13] Considering 10^3 periods in the future suffices, as for the largest discount factor the contribution of an additional period to the value of $S(k)$ is neglible for periods more than 10^3 periods ahead.

[14] All concomitant standard errors are of the order 10^{-5} or less.

	Per-period detection probabilities, p_t					
δ	**0%**	**0%-5%**	**0%-10%**	**0%-15%**	**0%-20%**	**0%-25%**
1.00	∞	20.00	10.00	6.67	5.00	4.00
		36.82	17.59	11.15	8.15	6.27
0.99	100.00	16.81	9.17	6.31	4.81	3.88
		27.02	15.12	9.87	7.49	5.99
0.98	50.00	14.49	8.47	5.99	4.63	3.77
		20.86	13.02	9.17	6.81	5.56
0.97	33.33	12.74	7.87	5.70	4.46	3.67
		17.16	11.72	8.42	6.45	5.39
0.95	20.00	10.26	6.90	5.19	4.17	3.48
		12.86	9.22	7.18	5.70	4.75
0.90	10.00	6.90	5.26	4.26	3.57	3.08
		7.86	6.38	5.35	4.55	3.97
0.75	4.00	3.48	3.08	2.76	2.50	2.29
		3.62	3.29	3.02	2.80	2.57
0.50	2.00	1.90	1.82	1.74	1.67	1.60
		1.92	1.85	1.78	1.72	1.65

Table 1: Maximum number of firms that sustains internal cartel stability for a symmetric homogeneous goods Bertrand oligopoly; each first cell entry is the theoretical upper bound, each next cell entry relates to normally distributed per-period detection probabilities with truncated support.

3.1 The stationary case

To grasp further the economics at work underlying the ICCs (7) the special case of the stationary supergame is considered. Note that (7) requires for every future moment in time an assessment of compliance payoffs and deviation payoffs in any future period. For stationary supergames the number of assessments reduces significantly since for every period t only one particular deviation period has to be considered.

Lemma 2 *The supergame is stationary if, and only if,* $p_t = p \in (0,1) \subset \mathbb{R}$ $\forall\ t$.

Proof. See Appendix 3. ■

According to Lemma 2 the game is stationary if detection probabilities are identical for each period. The stochastic part of the model then boils down to an infinite sequence of independent identical Bernoulli trials. Cartel-discovery probability (1) then simplifies to:

$$P_t(k) = 1 - (1-p)^{k+1}. \tag{8}$$

All ICCs (7) accordingly boil down to:

$$\frac{\pi^{defect} - \pi^c}{\pi^c - \pi^N} \leq \frac{\delta(1-p)}{1-\delta(1-p)}. \tag{9}$$

For the standard text book discount factor $\delta = (1+r)$, $r \in \mathbb{R}_+$, it reads as:

$$\frac{\pi^{defect} - \pi^c}{\pi^c - \pi^N} \leq \frac{1-p}{r+p}. \tag{10}$$

For the stationary case the impact of introducing a per-period detection probability on the minimum discount factor for internal cartel stability does not depend on specific profit levels:

Proposition 3 *In case of constant per-period detection probabilities* $p \in (0,1)$ *the necessary percentage increase of the discount factor for internal cartel stability equals* $100 \times p/(1-p)$.

Proof. From (9) let $\delta|_{p=0} = \delta = \Omega/(1+\Omega)$ and $\delta|_{p>0} = \delta\prime = \Omega/(1-p)(1+\Omega)$, where $\Omega = \left(\pi^{defect} - \pi^c\right)/\left(\pi^c - \pi^N\right)$, from which follows that $(\delta\prime - \delta)/\delta = p/(1-p)$. ■

The effect of detection probabilities on internal cartel stability is readily illustrated. For instance, the example of homogeneous goods with Bertrand competition among equally efficient firms yields an internally stable cartel of 4 firms for $\delta = 0.75$. Introducing $p = 0.25$ implies that this discount factor has to increase by 100/3 percent to $\delta = 1.00$ for the cartel to remain stable internally (see also Table 1).

4 Changing detection probabilities

Although from the onset a cartel could be internally stable, this could change if $f(x; \boldsymbol{\theta}(t))$ changes. There are many occurrences thinkable that would trigger such change; adjustments in antitrust law providing the authorities with more effective tools, changing political attitudes towards the need for strict antitrust enforcement (Ghosal, 2004), evolving structural market characteristics, and so on. The effect of a change in $f(x; \boldsymbol{\theta}(t))$ on internal cartel stability is also important to identify as $f(x; \boldsymbol{\theta}(t))$ can be seen as a policy instrument; at the end of the day the efforts of antitrust authorities determine the per-period detection probabilities. The next proposition summarizes:

Proposition 4 $\forall\ t > 1$ *an increase in* $p_t \in (0,1) \subset \mathbb{R}$ *reduces the domain for which the strictest ICC for internal cartel stability is not binding.*

Proof. $\forall k^* > 0\ \exists\ d^* > 0 \mid 1+d^* = k^*$. Then note that $\partial S(1+d^*)/\partial p_t < 0$ $\forall\ t > 1$. ■

According to Proposition 4 an increase *in any* per-period detection probability reduces the likelihood that the cartel is stable internally. Existing cartels could then fall apart while potential cartels might not be formed at all. In the special case of detection being a certain event in some (future) period no cartel can survive nor will be formed.

It is important to note that this change in $f(x; \boldsymbol{\theta}(t))$ is not to be attributed to time passing by, but to an exogenous change in f and/or $\boldsymbol{\theta}(t)$. For instance, initially it might be that $f(x; \boldsymbol{\theta}(t)) \sim UNIF(0, 0.05)$, but at some stage the antitrust authorities are better able to track down illegal cartels such that $f(x; \boldsymbol{\theta}(t)) \sim UNIF(0, 0.1)$. According to Proposition 4 *if only one* per-period detection probability increases the internal stability of the cartel is reduced.

The empirical work of Ghosal and Gallo (2001) could be interpreted as supporting the prediction of Proposition 4. They find that the number of antitrust cases initiated by the US Department of Justice is positively related to their extent of funding. A first explanation is, of course, that more funding means that more cases can be considered. A second explanation is that increased funding enhances the per-period detection probabilities. According to Proposition 4 this would lead to the breakdown of an additional number of cartels, and all these shatters come with relative large price movements thence attracting the attention of the antitrust authorities (see also footnote 9).

At face value Proposition 4 implies that all antitrust authorities have to do is to increase the detection probability for some future period. One way of establishing that would be to concentrate all efforts on one industry such that prosecution of potential cartels within this industry will be successful, to announce this success widely, and to move on to the next industry. This way all industries will be visited sometime in the future, and cartel detection, if applicable, would be quite a certain event. As a result for all industries some future per-period detection probability increases thereby reducing the internal stability of all existing cartels.

However, the actual effect on the internal stability of a cartel of increasing some future per-period detection probability is likely to be small, unless future detection approaches to be a certain event. The results in Table 1 suggest that incremental increases in per-period detection probabilities have a small to modest effect compared to the effect of introducing a per-period detection probability as such. For instance, for $\delta = 0.95$ the maximum number of cartel members drops from 20 to about 10 if a per-period detection probability of 5% is introduced, while increasing this probability from 15% to 20% reduces the maximum number of cartel members by one only. Accordingly, rather then trying to increase future per-period detection probabilities for all industries by sequentially putting all efforts into one industry, antitrust authorities should diversify their efforts such that for every industry there is some positive probability of cartel detection in every period.

4.1 Increasing per-period detection probabilities

An interesting special case is where per-period detection periods gradually increase over time. For instance, in many jurisdictions enforcement of antitrust laws is of recent date such that there is ample scope for the respective antitrust authorities to improve upon the effectiveness of their workings. Likewise, jurisprudence could have been built up that facilitates future convictions in comparable cases.

Within the framework developed here these examples imply that per-period detection probabilities are weakly increasing in time, that is, $f(x; \boldsymbol{\theta}(t))$ is such that $\forall\ i > j$:

$$p_{t+i} > p_{t+j}. \tag{11}$$

To consider this situation first observe:

Lemma 5 *The situation of weakly increasing per-period detection probabilities converging to* $p^* \in [0,1)$ *corresponds to the stationary version of the supergame with* $p = p^*$*.*

Proof. If (11) holds we have that $\forall k > 0$ $\partial S(k)/\partial k < 0$; ICC (7) is most binding for the largest possible k, that is, for the limit of $\mathbf{p} = \{p_1, p_2, ...\}$. Hence, if $\sup \parallel \mathbf{p} \parallel = p^* < 1$, the condition for internal cartel stability is given by (9) with $p = p^*$. ■

ICC (9) thus implies that the higher is the limit to which per-period detection probabilities converge, the lower is the likelihood that the cartel is internally stable. Yet, even if per-period detection probabilities are increasing in time but to a limit below 1, breaking down of the cartel is not a certain event (which is, of course, also illustrated by every first cell entry in Table 1). Also, Proposition 3 readily applies: the minimum discount factor for internal cartel stability increases with $100 \times p^*/(1-p^*)$ percent in case of increasing per-period detection probabilities converging to p^*.

5 Fine payments

To complete the analysis fine payments are considered, as in practise if a cartel is broken up because antitrust authorities have discovered it all former cartel members typically have to pay a fine $F \in (0, \overline{F}]$ once. The upper bound on fine payments is given by law and for an individual firm usually is defined as a percentage of gross annual per-firm revenue in the last year of the cartel's existence. The actual fine imposed on an individual firm depends on specific circumstances, including the cumulative damages caused by the cartel (either to industry or to consumers) and whether or not the firm has initiated the cartel as such.[15]

Incorporating these penalties into the model yields at the beginning of period t as expected fine payment in period $t + k$ (see also Figure 2):

$$F_t(k) = \begin{cases} p_t F & if\ k = 0, \\ [1 - P_t(k-1)]\, p_{t+k} F & otherwise. \end{cases} \quad (12)$$

In the long run the expected fine payment during any period is zero as the cartel is no longer expected to exist, that is, $\lim_{k\to\infty} F_t(k) = 0$.

[15] See Article 15 of EU Council Regulation 17 for details of the EU practise; see Chapter 2 of the DoJ Antitrust Resource Manual (the "Sherman Act") for details of the US practise.

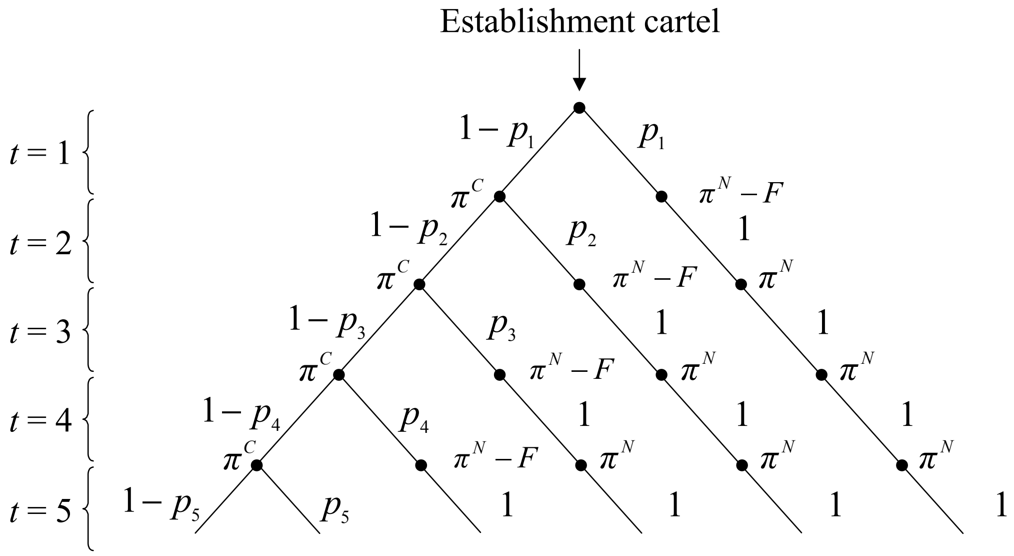

Figure 2: Development over time of expected cartel adherence payoffs with fines.

But what matters for internal cartel stability is the expected overall fine payment, defined as $\Gamma_t(k) = \sum_{i=0}^{k} \delta^i F_t(i)$. To consider the composition of $\Gamma_t(k)$ first note the relation between the expected per-period fine payment and per-period detection probabilities:

$$\left.\frac{\partial F_t(k)}{\partial p_{t+j}}\right|_{k>0} = \begin{cases} -\prod\limits_{l=0,l\neq j}^{k-1}(1-p_{t+l})p_{t+k}F < 0, & j < k, \\ \prod\limits_{l=0}^{k-1}(1-p_{t+l})F > 0, & j = k, \\ 0, & j > k. \end{cases} \tag{13}$$

In case $k = 0$ only the second and third part of (13) apply, having the same sign for the respective partial derivatives. Accordingly, there is a one-to-one and positive relation between $F_t(k)$ and p_{t+k}; an increase in any per-period detection probability increases the expected fine for that period.

At the same time, an increase in detection probability in period t reduces the expected fine payment in any later period $t + j$, $j > 0$, simply because the probability of reaching that future period as a cartel is reduced. The net effect of these two opposite forces on the expected overall fine payment is positive; increasing *any* per-period detection probability raises the overall expected fine payment, as shown in the following lemma:

Lemma 6 *If* $\delta \in [0,1] \subset \mathbb{R}$, $p_t \in (0,1) \subset \mathbb{R}$, $F \in (0, \overline{F}) \subset \mathbb{R}_+$, $k \in \mathbb{N}$ *then* $\forall t > 1$ *and* $j \in \mathbb{N}$ *we have that* $\partial \Gamma_t(k)/\partial p_{t+j} > 0$, *where* $\Gamma_t(k) = \sum_{i=0}^{k} \delta^i F_t(i)$.

Proof. See Appendix 4 ■

The ICCs for internal cartel stability now follow. As before, defection during period t is assumed not to affect the probability that the cartel is discovered in the same period. This means in particular that also during the defection period there is a possibility that the defecting firm has to pay the fine F. Comparing then $V_t^{defect}(d) - \Gamma_t(d)$ with (3), realizing that the expected discounted value at the beginning of period t of adherence during period $t + k$ equals $v_t^c(k) - F_t(k)$, yields as ICCs (see also Appendix 1):

$$\frac{\pi^{defect} - \pi^c}{\pi^c - \pi^N} \leq S(t + d) - \widetilde{F} \sum_{i=1}^{\infty} \delta^i p_{t+d+i} \prod_{j=1}^{i-1} (1 - p_{t+d+j}) = S_F(t + d), \quad (14)$$

$t = 1, 2, ...,$ $d = 1, 2, ...,$ where $\widetilde{F} = F/(\pi^c - \pi^N)$. Note that a straightforward proof along the lines of that in Appendix 2 establishes that the minimum over k of $S_F(k)$ exists and that it is unique, provided that $\overline{F}$ is such that $S_F(k) > 0$, $\forall\ F \in (0, \overline{F}) \subset \mathbb{R}_+$.

The ICCs (14) are numerically illustrated in Table 2 for $F = \gamma\pi^c$, with $\gamma = 0.1$ corresponding to the maximum fine of 10% of gross cartel profits. Comparing these results with those in Table 1 clearly illustrates that fines reduce the internal stability of a cartel; the number of firms for which any cartel is stable internally reduces with prospective fine payments. These illustrations are formalized in the next proposition:

Proposition 7 *An increase in F reduces the domain for which the strictest ICC for internal cartel stability is not binding.*

Proof. Let $S_F(k^*) = \min_{\{k\}} S_F(k)$, $k > 1$. Then $\exists\ d^* > 0 \mid 1 + d^* = k^*$, and $\partial S_F(k^*)/\partial F = \partial S(k^*)/\partial F - \frac{1}{(\pi^c - \pi^N)} \sum_{i=1}^{\infty} \delta^i p_{1+d^*+i} \prod_{j=1}^{i-1} (1 - p_{1+d^*+j}) < 0\ \forall$ $p_t \mid p_t \in (0,1) \subset \mathbb{R}$. ■

For an individual firm increasing fine payments thus reduces the likelihood that a collusive agreement is profitable in an expected sense. Larger fine payments reduce both the expected single-period defection profits and the expected single-period collusive profits. As a result the ICCs become more strict since an increase in fine payments does not affect the expected per-period noncooperative Nash profits.

	Per-period detection probabilities, p_t					
δ	**0%**	**0%-5%**	**0%-10%**	**0%-15%**	**0%-20%**	**0%-25%**
1.00	∞	18.00 40.33	9.00 19.98	6.00 13.38	4.50 9.72	3.60 8.01
0.99	100.00	15.14 28.42	8.27 16.55	5.68 11.96	4.33 9.11	3.50 7.25
0.98	50.00	13.07 22.41	7.64 14.52	5.40 10.82	4.18 8.41	3.40 6.99
0.97	33.33	11.50 18.38	7.11 12.95	5.12 9.73	4.03 7.96	3.31 6.50
0.95	20.00	9.28 13.48	6.24 10.27	4.70 8.14	3.77 6.85	3.15 5.89
0.90	10.00	6.28 8.16	4.79 6.87	3.87 5.90	3.25 5.26	2.80 4.63
0.75	4.00	3.22 3.71	2.85 3.48	2.55 3.25	2.31 3.05	2.11 2.89
0.50	2.00	1.81 1.95	1.72 1.90	1.65 1.85	1.58 1.81	1.52 1.77

Table 2: Maximum number of firms that sustains internal cartel stability for a symmetric homogeneous goods Bertrand oligopoly with a fixed fine of 10% of gross cartel revenue; each first cell entry is the theoretical upper bound, each next cell entry relates to normally distributed per-period detection probabilities with truncated support.

At the same time the results in Tables 1 and 2 indicate that the effect of a fixed fine of 10% of gross cartel profits is rather small; in all cases the number of firms that sustains internal cartel stability is reduced by less than two. In the particular case of a stationary supergame the maximum number of cartel members drops by $10 \times \delta p$ percent for the cartel to remain stable internally.

Note further that the strictest ICC in (14) implicitly defines the penalty that erases every current and future cartel, being the fine that follows from $S_F(k^*) \leq 0$. For the stationary case an elegant rule emerges (which of course also applies in case of weakly increasing per-period detection probabilities converging to $p^* \in (0,1)$):

Proposition 8 *In case of constant per-period detection probabilities* $p \in (0,1)$ *the minimum fixed fine that makes the ICC for internal cartel stability always binding is* $100 \times (1-p)/p$ *percent of incremental cartel profits.*

Proof. For $p_t = p\ \forall t > 0$, we have that $S_F(k) = \delta[1 - p(1+\widetilde{F})] \Big/ [1 - \delta(1-p)]$, with $\widetilde{F} = F/\left(\pi^c - \pi^N\right)$. Solving $S_F(k) \leq 0$ then yields $F \geq \left(\pi^c - \pi^N\right)(1-p)/p$. ■

Based on estimates of per-period detection probabilities a simple rule emerges from Proposition 8. If these probabilities are for instance 17% the fine after detection should be almost 5 times the incremental profits due to collusion for a cartel never to be stable internally. In absence of an antitrust authority the necessary fine becomes infinitely large.

In sum, both increasing prospective fine payments and increasing per-period detection probabilities reduce the domain for which the strictest ICC is not binding. In this sense fine payments and detection probabilities are substitutable instruments for breaking down illegal cartels. This is also illustrated with Proposition 8 as the necessary fine is decreasing in p. At the same time the two instruments are complementary in that either supports the effectiveness of the other. From the proof of Proposition 7 it is immediate that $|\partial^2 S_F(k^*)/\,\partial F \partial p_t| > 0$; increasing any future per-period detection probability reduces more the domain for which the strictest ICC constraint is not binding, the larger is the prospective fine payment. Likewise, it is straightforward to show that $|\partial^2 S_F(k^*)/\,\partial p_t \partial F| > 0$ (see also the proof of Lemma 6); increasing prospective fine payments reduces more the domain for which the strictest ICC is not binding, the larger is any future per-period detection probability.

6 Conclusions

The theoretical literature is starting to address properly the possibility of cartel detection. This paper contributes to that literature by analyzing a supergame of collusion in which there is a per-period detection probability that can be different in any period. Numerical simulations of the ensuing nonstationary supergame indicate that for discount factors up to 10% per-period detection probabilities of little over 5% are needed to reduce the number of firms that can sustain a cartel by 50% or more.

The main result is that an increase *in any* per-period detection makes the ICC for internal cartel stability more strict. Although this would suggest that antitrust authorities are most effective in breaking cartels by investigating industries consecutively in great detail, numerical simulations show that introduction of a small per-period detection probability has a much greater effect on internal cartel stability than an incremental increase of this probability. Antitrust authorities should therefore diversify their limited resources over all industries such that each of these are confronted with a strictly positive detection probability.

A second result is that a situation of strictly non-decreasing per-period detection probabilities converging to some probability p^* corresponds to the stationary version of the supergame where all detection probabilities are constant. For this special case two elegant rules apply: internal cartel stability requires an increase of $100 \times p^*/(1-p^*)$ percent of the minimum discount factor, and a fine of $100 \times (1-p^*)/p^*$ percent of incremental cartel profits makes the ICC for internal cartel stability always binding.

As with any generalization of a widely used principle our analysis qualifies studies using that principle. Especially in the field of cartel formation in oligopolistic industries there is ample scope for applying the presently developed framework to specific applications of Friedman's trigger strategy that have ignored the possible detection of the cartel by the antitrust authorities.

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7 Appendix 1 Incentive compatibility

Comparing V_t^{ncc} with $V_t^{defect}(d)$ first yields:

$$\frac{\pi^{defect} - \pi^{ncc}}{\pi^{ncc} - \pi^C} \leq \frac{1}{1 - P_t(d)} \sum_{i=1}^{\infty} \delta^i \left[1 - P_t(d+i)\right].$$

Then note that $\left[1 - P_t(d+i)\right] / \left[1 - P_t(d)\right] = \prod_{j=1}^{i}(1 - p_{t+d+j})$.

8 Appendix 2 Proof of Lemma 1

(i) Suppose $\exists\, k > 0 \mid S(k) = \emptyset$, then $\sum_{i=1}^{\infty} \delta^i \prod_{j=1}^{i} (1-p_{k+j}) = \emptyset$, or $\prod_{j=1}^{i} (1-p_{k+j}) = \emptyset\ \forall\ i > 0$, given that $\delta \in [0,1] \subset \mathbb{R}$. Yet, if $p_t \in (0,1) \subset \mathbb{R}\ \forall\ t > 0$ then $\prod_{j=1}^{i} (1-p_j) \neq \emptyset$, a contradiction.

(ii) Repeated substitution yields:

$$S(k) = \sum_{i=1}^{l} \delta^i \prod_{j=1}^{i} (1-p_{k+j}) + \delta^l \prod_{j=1}^{l} (1-p_{k+j}) S(k+l),$$

$l \in \mathbb{N}$, $k \in \mathbb{N}^+$. Setting $S(k) = S(k+l)$ yields:

$$S(k+l) = \frac{\sum_{i=1}^{l} \delta^i \prod_{j=1}^{i} (1-p_{k+j})}{1-\delta^l \prod_{j=1}^{l} (1-p_{k+j})} = \sum_{i=1}^{\infty} \delta^i \prod_{j=1}^{i} (1-p_{k+j}) = S(k). \tag{15}$$

Equality (15) holds for $\lim_{p_t \to 0}\ \forall\, t > 1$. Yet, $\left|\partial LHS_{(15)}/\partial p_t\right| > \left|\partial RHS_{(15)}/\partial p_t\right|$ $\forall\ t > 1$, thus contradicting the claim that $S(k) = S(k+l)$ for $p_t \in (0,1) \subset \mathbb{R}$ $\forall\ t > 0$.

9 Appendix 3 Proof of Lemma 2

Stationarity requires (see e.g. Mas-Colell, 1995, p. 734):

$$V_t^{ncc} \geq V_t^{defect}(k) \Leftrightarrow V_t^{ncc} \geq V_t^{defect}(k'), \forall k \neq k'.$$

For $k' = 0$, condition (7) is:

$$\frac{\pi^{defect} - \pi^{ncc}}{\pi^{ncc} - \pi^C} \leq \frac{1}{1-p_t} \sum_{i=1}^{\infty} \delta^i \left[1 - P_t(i)\right].$$

Stationarity thus holds if, and only if, $\forall k > 0$:

$$\frac{1}{1-p_t} \sum_{i=1}^{\infty} \delta^i \left[1 - P_t(i)\right] = \frac{1}{1-P_t(k)} \sum_{i=1}^{\infty} \delta^i \left[1 - P_t(i+k)\right],$$

or:

$$\sum_{i=1}^{\infty}\delta^i\prod\nolimits_{j=1}^{i}(1-p_{t+j})=\sum_{i=1}^{\infty}\delta^i\prod\nolimits_{j=k+1}^{i+k}(1-p_{t+j}).$$

That is, $\forall i,k>0$ it must be that:

$$\prod\nolimits_{j=1}^{i}(1-p_{t+j})=\prod\nolimits_{j=k+1}^{i+k}(1-p_{t+j}).$$

For $i=1$ we have that $(1-p_{t+1})=(1-p_{t+k})$, which holds $\forall k>1$ if, and only if $p_t=p$, $\forall t$.

10 Appendix 4 Proof of Lemma 6

First note that $\Gamma_t(k)$ is the overall expected fine payment at the beginning of period t through period $t+k$. Then:

$$\frac{\partial\Gamma_t(k)}{\partial p_{t+j}}=\begin{array}{cc}\sum_{l=j}^{k}\delta^l\partial F_t(l)/\partial p_{t+j}, & j\leq n,\\ 0 & j>n.\end{array}$$

Since firms consider all future periods, restrict attention to $j\leq k$. Using (13) we have:

$$\sum_{l=j}^{k}\delta^l\frac{\partial F_t(l)}{\partial p_{t+j}}=\delta^l\prod_{l=0}^{j-1}(1-p_{t+l})F-\delta^{l+1}\prod_{l=0,l\neq j}^{j}(1-p_{t+l})p_{t+j+1}F-...-\delta^k\prod_{l=0,l\neq j}^{k-1}(1-p_{t+l})p_{t+k}F. \tag{16}$$

For (16) to be positive it is sufficient to show that it is positive for $\delta=1$. In that case (16) translates into:

$$\sum_{l=j}^{k}\delta^l\frac{\partial F_t(l)}{\partial p_{t+j}}=\prod_{l=0,l\neq j}^{k}(1-p_{t+l})>0.$$