“What’s on sale this week?” Pricing and advertising strategies in retailing

Master Thesis of Bart Voogt

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Chapter 1

Introduction

Supermarkets usually carry an assortment of thousands of goods. It is therefore not surprising that supermarkets only advertise a subset of their assortment. However, it is less clear why, for example, a supermarket advertises beer this week and bread in the next. In this paper we address two questions: what determines which and how many goods a supermarket advertises; why are advertised prices relatively volatile in comparison to non-advertised prices?

We approach these issues from the perspective of informative advertising.[1] Under the informative view, advertising functions as a way to convey information, like store's prices and location, to consumers. In this sense, informative advertising has a competitive nature, since firms try to attract consumers by advertising appealing prices. One promotional activity of this kind is known as leader pricing: firms offer discounts on advertised goods (leaders) and make (most of the) profit on non-advertised goods. The argument is as follows. Economies of scale in shopping results in firms having some monopoly power over the consumers who enter their store.[2] Consumers are aware that firms will try to exploit this power by charging high prices. Therefore, firms offer discounts on advertised goods, the leaders, to guarantee the consumers a certain amount of surplus when visiting the firm. Clearly, advertising functions

[1]The other main views on advertising are the persuasive and the complementary view. See Bagwell (2005) for a detailed survey.

[2]Consumers can, for example, face transportation cost, such that visiting another firm is costly. Another argument is that of convenience shopping: consumers prefer to visit only firm, because they do not want to stand in line for the cash-register multiple times, etc.

as a commitment device and aims at increasing store traffic. Once in the store, firms hope that consumers will also buy non-advertised goods, which are priced relatively high. Markets in which firms carry a large assortment of items, like supermarkets, are particularly suited for leader pricing strategies.

The analysis focuses on a relatively stylized advertising model. Two firms carry an assortment of two independent goods (neither substitutes nor complements). Each firm has the option to advertise zero, one or both goods. Consumers can visit one firm at maximum (convenience shopping) and, at first, only observe the advertised prices. They learn the non-advertised prices of a firm when visiting its store. The consumer population is homogeneous and each buyer desires to buy one unit of each good; the goods are valued equally.

In chapter 4 we characterize the equilibria and show that advertised prices are random and that firms either mix between advertising zero and one good or mix between advertising zero, one or both goods. The homogeneity of the consumer population is crucial for our results, because it implies that a firm can capture the entire market by slightly undercutting its rival's price. This and the fact that advertising is costly leads to the result that, in equilibrium, firms use a mixed strategy to determine their advertising intensity and the prices of the goods they advertise. These results contrast earlier results obtained in a setting where firms are located at the opposite ends of a Hotelling line; since consumers are less price sensitive in such a case it is more likely that the equilibrium is in pure strategies.[3] The motivation for our assumption is that supermarkets are often located close to each other in shopping centres.

Another result is that non-advertised prices are constant at the monopoly level. The reason for this is, like stated earlier, that firms have monopoly power over the consumers who enter their store. Since the firms have committed themselves to their advertised prices, only the non-advertised prices can be set at the monopoly level. The implication of this result is that consumers only expect to get surplus on the advertised goods. The strategy of the consumers is therefore simple: they visit the store that offers them the highest surplus on the advertised goods. When firms do not advertise at all or when firms offer the same amount of surplus, then the consumers are indifferent and choose the store to visit at random.

[3]See for example Lal & Matutes (1994).

In equilibrium, consumers prefer to visit the firm with the highest advertising intensity. This implies that advertising does indeed increase store traffic. The intuition behind this result is as follows. Firms use advertising as a commitment-device to ensure the consumers a certain amount of surplus when visiting their store. When a firm doesn't advertise at all it cannot guarantee the consumers any surplus, such that consumers prefer to visit a firm that advertises over a firm that doesn't advertise. The main reason for firms to advertise both goods instead of one is that by advertising only one good the firm is only able to guarantee a certain maximum amount of surplus, namely the reservation price of the advertised good. When a firm advertises both goods instead, it can potentially guarantee the consumers more surplus, namely up to the reservation prices of both goods. Clearly, the higher the advertising intensity of a firm the more potential it has to attract consumers to its store. Since advertising is costly, firms only invest heavily in advertising if they intend to use their relatively high potential to attract consumers by setting relatively low (advertised) prices. Therefore, average prices are decreasing in advertising intensity.

Another implication of the strategy of the consumers is that they are indifferent with respect to which good is advertised; this is because they maximize the surplus obtained from their whole shopping-list rather than the surplus derived from individual goods. Due to this indifference, firms themselves are also indifferent with respect to which good they advertise.

These results, which are derived below in Chapters 3 to 4, are interesting because they help answer why supermarkets advertise beer this week and bread in the next; why advertised prices are dispersed; why non-advertised prices are much less volatile than advertised ones; why firms advertise more heavily in one week than in the other. In chapter 2 we relate our findings to the relevant empirical and theoretical literature.

Chapter 5 is devoted to study whether firms have a preference for advertising a good with a relatively low value (good L) over a good with a relatively high value (good H) and vice versa. This is particularly interesting, because the empirical literature suggests that supermarkets tend to promote branded-goods rather than own-brand goods. Our results indicate that firms are either indifferent between advertising good L and good H or that they prefer to advertise good H. The intuition behind this result is that goods with a relatively high value have a higher potential to attract consumers to visit a store. When a firm plans to offer relatively large discounts it prefers to advertise good H, because the maximum discount a firm can offer on a good is its reservation price and the reservation price of good H is higher than that of good L. When, however, a firm plans to offer relatively low discounts it is indifferent between advertising good L and good H because these discounts can be offered on both types of goods.

In chapter 6 we provide a discussion of how our results would be modified in more general settings. We argue that the model can be enriched by introducing elastic demand and by allowing consumers to visit more than one firm. Chapter 7 concludes and, finally, in an appendix we give a summary of our results when we relax the assumption that unit selling costs are equal to zero. We show that, in equilibrium, firms can charge a price below unit selling costs (loss-leader pricing).

Chapter 2

Literature review

This paper is mainly related to the literature on leader pricing. In this literature there are two theories of leader pricing. In one of those theories leader pricing functions as a signaling device. Nelson (1974) suggests that firms advertise low prices to signal low costs. The idea is that consumers rationally expect that a firm with low costs will charge relatively low prices for its unadvertised goods as well. In this paper we focus on the other theory of leader pricing, namely the theory that leader pricing functions as a commitment device.

To the best of our knowledge, Hess and Gerstner (1987) were the first to study loss leader pricing formally in a model.[1] They construct a two-period game in which firms have an assortment of one shopping good and a variety of impulse goods ("products bought on sight without price comparisons across stores"). Firms use the shopping good as a (loss) leader to attract consumers to their stores. Consumers visit only one store per period, such that firms have monopoly power over the consumers visiting their store. Firms exploit this power by charging relatively high prices for the impulse goods. The pricing of the shopping goods is really competitive, because it is assumed that consumers are aware of the price of the shopping good of each firm. The result is that shopping goods are priced below unit selling costs. Therefore, firms have an incentive to reduce their stock of shopping goods. Consumers are offered rain-checks when the store runs out of stock.

Rain-checks are another form of commitment: the (advertised) price of the shopping good

[1] Loss-leader pricing is a form of leader pricing in which the leader is sold below unit selling costs.

ensures the consumers of an attractive deal when visiting a firm, the rain-check ensures that consumers will actually be able to close this deal (now or in the future). Hess and Gerstner argue that firms deliberately run out of stock to increase future store traffic. The idea is that those consumers who return to the store with a rain-check (in the next period) visit the store twice and therefore buy more impulse goods than when they visit the store only once.

Lal & Matutes (1994) relax the assumption of impulse goods by allowing consumers to decide which store to visit based on the surplus derived from the purchase of an assortment of goods. Moreover, in their model consumers are allowed to visit more than one store. They assume that, prior to visiting a store, consumers are uninformed about the prices of firms unless advertised. Unlike in Hess and Gerstner, in which it is assumed that one particular good is the leader, firms have the option to decide how many and which goods they advertise.

Lal and Matutes's model features two firms which are located on the opposite ends of a Hotelling line. Both firms carry an assortment of two independent goods (neither substitutes nor complements). Firms can advertise no good, one good or both goods. Advertising costs increase linearly in the number of goods a firm advertises. Consumers have identical unit demands for both goods and are uniformly distributed over the Hotelling line. In equilibrium both firms either advertise one and the same good or they advertise both goods. Non-advertised prices are at the monopoly level, while advertised prices are lower and potentially below unit selling costs. The intuition behind this result is that firms have an incentive to advertise the same good to avoid that consumers shop around and only buy the relatively cheap advertised goods.[2]

Given the identical pricing and advertising strategies of the firms, consumers have no incentive to shop around and they buy both goods at the same store. Consumers have rational expectations and realize that firms will try to exploit them when they enter the store, because at that moment their transportation costs become sunk. The role of advertising as a commitment device is clear: firms guarantee consumers a certain amount of surplus to cover their transportation costs through advertising. Without this commitment we would

[2]Since firms choose their advertising strategies simultaneously it is unclear how firms will coordinate on a particular equilibrium. In such coordination games there is usually a third, more plausible, equilibrium in mixed strategies. However, Lal & Matutes do not allow for mixed strategies.

have the 'Diamond result' and consumers would not even bother to shop in the first place.[3]

In an extension of their model Lal & Matutes relax the assumption that consumers' willingness to pay is the same for both goods. By doing so they can study whether high or low reservation price goods are offered as loss leaders. Like in the basic model, firms have an incentive to advertise the same good. Lal & Matutes refrain from characterizing the equilibria completely, but show that there is an equilibrium in which both firms advertise the good with the lower reservation price and an equilibrium in which both firms advertise the good with the higher reservation price. Denoting by H (L) the reservation price for the good with the higher (lower) reservation price and by c is the unit-selling cost of either of the goods, in the former equilibrium firms charge a price of $c - H$ for the advertised good and a price of H for the non-advertised good, while in the latter equilibrium firms charge a price of $c - L$ for the advertised good and a price of L for the non-advertised good. It is clear that in both equilibria consumers pay the same amount to buy both goods, namely c. The existence conditions placed on these equilibria are the same, so it can not be argued that one good is more likely to be advertised than the other. However, Lal & Matutes argue that it is more likely that the good with the lower reservation price is the loss-leader, because $c - L > c - H$. The good with the lower reservation price is advertised against a relatively low price and it is therefore more likely that this price falls below unit selling costs. Clearly, one should distinguish between the probability that a good is advertised and the probability that a good is a loss-leader.

Rao and Syam (2001) modify the framework of Lal & Matutes by allowing firms to use mixed strategies. They show that in equilibrium firms follow a mixed strategy of advertising one good or the other and that firms also offer discounts on the non-advertised goods. The mixed-strategy equilibrium yields higher profits than the equilibrium in Lal & Matutes. The reason for this is that although non-advertised goods in Lal & Matutes are priced at the monopoly level, competition on the advertised good is severe. By contrast, competition on the advertised good in the mixed-strategy equilibrium is reduced because the firms are un-

[3] Diamond (1971) shows that (in a model without advertising) when search costs are bounded above zero, the unique Nash-Equilibrium is the monopoly price. The idea behind his result is that consumers rationally expect firms to charge the monopoly price for their goods so there do not exist gains from search.

certain about which good its rival advertises. Another effect of this uncertainty is that firms run the risk that they do not advertise the same good and that consumers take advantage of this by visiting both stores and buying only the relatively cheap advertised goods. Rao and Syam show that the former effect is stronger than the latter leading to higher prices on average (in comparison to Lal & Matutes). Furthermore, firms offer discounts on non-advertised goods (unadvertised specials) to prevent consumers from shopping around when firms happen to advertise different goods.

Ellison (2005) constructs a model in which the goods are complements rather than independent. Two firms, located on opposite ends of a Hotelling line, carry an assortment of a base good and an add-on. The add-on only provides utility when consumed together with the base good. An example of such a business is an appliance store which sells personal computers (base good) and offers extended warranties in store (add-on). Besides their location on the Hotelling line consumers can be split up in two groups: one group of consumers is more likely to buy add-ons than the other group. This heterogeneity in the consumer population turns the model into one of price discrimination: firms offer the base good and a bundle, which consists of the base good and the add-on. The base good is aimed at the group which is not likely to buy the add-on, while the bundle, which is of a higher quality, is aimed at the other group. The strategy of the firms is familiar: firms attract consumers by charging an attractive price for the base good and hope to make a profit by pushing the high priced bundle when consumers enter their store. Ellison argues that firms intentionally create an adverse selection problem to limit competition. By setting a relatively low price for the base good a firm attracts a consumer pool with a disproportionate share of consumers who are not likely to buy the (profitable) bundle.

Gerstner and Hess (1990) consider a model in which firms use a "bait and switch" strategy. The idea behind this strategy is that firms advertise an attractive low quality good and try to convince consumers to buy a good with a higher quality and price when they enter their store. It is clear that for this strategy to be effective the goods in the assortment of a firm should be substitutes. There are two important differences between a model with substitutes and models with independent or complementary goods. First, in the former models firms do not plan to actually sell the loss-leader. And secondly, the prices of the substitutes depend highly on each other, because firms want to charge prices such that consumers are indifferent between buying the lower quality and the higher quality good. When the price difference becomes too large then consumers buy the low quality good and a too small price difference results in lower profits.

Gerstner and Hess show that low priced advertised goods are likely to be under-stocked to increase the probability that consumers buy the higher priced goods. They argue, however that this strategy is not necessarily harmful to consumers, because the competition effect leads to relatively low prices for the advertised goods and provides firms with an incentive to use in-store promotions to prevent frustrated consumers from taking their business elsewhere.

As Ellison (2005) notes, there is surprisingly little empirical work on loss-leader pricing. The paper of Walters and McKenzie (1988) seems to be the standard reference in the marketing literature. In contrast to the theoretical findings, he finds that loss-leader pricing does little to increase profits and store traffic. Our paper is able to explain these results from a theoretical point of view. We show that firms follow a mixed strategy to determine how many goods they advertise and at which prices they advertise these goods. A characteristic of equilibria in mixed strategies is that the expected profit is equal for all possible outcomes of these strategies. The expected profit of a low advertising intensity and a high advertising intensity is therefore equal. The idea behind this result is that a firm with a low (high) advertising intensity will attract a small (large) group of consumers, but has large (small) margins on its goods. This explains why profits are not affected by loss-leader pricing.

In their regression Walters and McKenzie use a dummy variable for loss-leaders and only goods offered at a discount of at least 15% are included as a loss-leader. Our model shows, however, that not the individual prices of goods are important, but rather the sum of all the prices on the shopping-lists of consumers. To test this result properly, we should regress the sum of all the discounts in an advertisement against store traffic. Suppose, for illustrative purposes, that one firm offers one good at a discount of 15%, while another firm offers this good and 100 other goods at a discount of 14%. Clearly, the effect of loss-leader pricing on store traffic cannot be correctly measured by using the methods of Walters and McKenzie in this case.

Our paper is mostly related to the work of Lal & Matutes. The framework we use is richer in the fact that we allow for mixed strategies. The mixed strategy equilibria that we derive are able to explain why advertisements and prices in retailing appear to be random. Another important difference is that we do not allow for negative prices. We argue that this restriction is reasonable and that it has important implications on the advertising strategies. The idea is that without a restriction on prices, firms can offer a relatively large discount on one good (leading to a negative price) instead of two relatively small discounts on two goods. Both offers are equally attractive to consumers, but the former offer leads to a saving on advertising costs, because the firm only has to advertise one good instead of two. In chapter 5 we discuss this issue more extensively.

Chapter 3

The model

Consider a market with two profit-maximizing firms, which carry an assortment of two independent goods. Let us assume that unit selling costs are equal to zero.[1] Initially, consumers know the location of the firms but they ignore the prices they charge for the two goods. The firms can advertise to attract consumers, but advertising is costly. We consider the following simple advertising technology: a firm can advertise one good at a cost of $A > 0$, or two goods at a cost of γA, with $1 < \gamma \leq 2$. This advertising technology assumes that there are (weakly) increasing returns to advertising. It is assumed that the advertisements reach all consumers. Notice that advertisements inform consumers about prices.

Consumers, whose mass is normalized to 1, value each of the products at v, with $v > A$, and hold inelastic demands, that is, they buy at most 1 unit of each product. We assume that consumers can visit a first store at no cost; however, consumers will not search beyond the first store due to the existence of sufficiently large search-costs. As a result, consumers will visit one store at maximum (convenience shopping).[2]

The timing of moves is as follows. First the firms simultaneously choose their prices and whether they will advertise one good, two goods, or no good at all. Then, consumers observe the advertised prices, form expectations about the non-advertised prices and decide which

[1] This assumption is not essential for the results of our paper. In the appendix we give the characterization of the equilibria when we relax this assumption. We show that firms can possibly charge prices below unit selling costs (loss-leader pricing).

[2] The assumption that consumers visit the first store at no cost is quite standard in the literature on search and advertising.

store to visit. Once a consumer walks into a store, he/she learns the prices of non-advertised products and decides which products to buy. We will focus on symmetric equilibria.

Throughout the analysis, we will use some tie-breaking assumptions. We'll assume that, everything else equal, a consumer will visit the store which advertised the largest number of products. The rationale behind this assumption is that advertising increases awareness so firms that advertise more intensively should be visited more often than firms which are less active in the advertising market.[3] In case of a tie in advertising and prices, the consumer will pick a store at random.

[3]This price-tie breaking rule has no bearing on our results. As we will see below, one can use price deviations that differ in a small enough ε from the proposed deviations to support the same equilibria.

Chapter 4

Characterization of equilibria

We first introduce some notation. Let λ_{ij} denote the probability that firm i advertises j products, $i = A, B$ and $j = 0, 1, 2$. In our analysis, what will be important for consumer decision-making is the sum of the prices of the items sold by a firm. Let p_{ij}^{A} then denote the *total* price firm i charges for its advertised good(s), $i = A, B$ and $j = 0, 1, 2$, with j representing the number of goods which the firm advertises. Finally, the *per-product* price firm i charges for its non-advertised good(s) is denoted by p_{ij}^{NA}, $i = A, B$ and $j = 0, 1, 2$, with j representing the number of goods promoted by the firm. First we'll derive an auxiliary result about prices that are not advertised.

Lemma 1 *Non-advertised prices will always be set at the monopoly-level:* $p_{ij}^{NA} = v$.

Proof. First of all, notice that since marginal costs are equal to zero, firms are always willing to sell their products. Therefore, firms do not have an incentive to set a price above v. Secondly, consumers do not have the option to shop around, so that an individual firm holds in fact a monopoly position over the consumers who enter its store. Clearly, firms will not set a $p_{ij}^{NA} < v$, because once at the store consumers will buy the non-advertised product if $p_{ij}^{NA} \leq v$. ■

Proposition 1 *There exists no equilibrium in pure advertising strategies.*

The proof of this result follows from the next three lemmas.

Lemma 2 *There exists no equilibrium where firms do not advertise at all,* $\lambda_0 = 1$.

Proof. Given lemma 1 we have the following pricing strategy: $p_{i0}^{NA} = v$. Consumers will now pick a store at random, resulting in equilibrium profits of: $\pi_i^* = \frac{1}{2} \times 2v = v$. A firm might deviate by advertising one product at a price of v, such that he'll capture the entire market.[1] This deviating strategy, $\lambda_{i1} = 1$ with $p_{i1}^A = p_{i1}^{NA} = v$, leads to profits of: $\pi_i = 2v - A$. This deviation is profitable if $v \leq 2v - A$, which can be rewritten as $v \geq A$, which follows from the assumption: $v > A$. ■

Lemma 3 *There exists no equilibrium where firms advertise both products,* $\lambda_2 = 1$.

Proof. In this case consumers will be fully informed about all prices when they have to make their shopping-decision. Therefore, we'll simply get the Bertrand-result, with $p_{i2}^A = 0$ and equilibrium profits: $\pi_i^* = -\gamma A$. Clearly, this cannot be an equilibrium. ■

Lemma 4 *There exists no equilibrium, where firms advertise one product,* $\lambda_1 = 1$.

Proof. Given lemma 1 we know that $p_{i1}^{NA} = v$. The consumers know that they will not get any surplus on the non-advertised product. Therefore, they'll visit the store with the lowest advertised price. Note that it doesn't really matter which product is advertised, but only at which price it's advertised. Given consumers behavior, firms competition leads to the Bertrand-result for the advertised prices: $p_{i1}^A = 0$. We can now calculate equilibrium profits: $\pi_i^* = \frac{1}{2}v - A$.[2]

Notice that, since firms have the option not to advertise at all, profits must be nonnegative, which implies that $v \geq 2A$ for the proposed equilibrium to exist. Now let's consider a deviation in which a firm advertises both products at a total price of v: $\lambda_{i2} = 1$ with $p_{i2}^A = v$. Since the total price is the same, the deviating firm attracts all consumers, $\pi_i = v - \gamma A$.[3] Due to economies of scale in advertising, it's costs, however, will be less than double. As a

[1] In case of a tie, the consumer will pick the firm that advertised the most products. Without this tie-breaking rule we could have simply used $v - \varepsilon$ instead, which, for small ε gives the same result.

[2] Notice that advertising is used as a commitment device and that, in this case, firms would like to commit to a price: $p_{i1}^A < 0$, in order to capture the entire market. However, since negative prices are not allowed, firms can only commit themselves to lower prices by increasing their advertising (both products instead of one).

[3] Otherwise we could simply assume that the firm advertises a total price of $v - \varepsilon$.

result, the deviation is profitable if $\frac{1}{2}v - A \leq v - \gamma A$, or $v \geq 2A(\gamma - 1)$. Since $v \geq 2A$, we can see that only in the non-generic case that $\gamma = 2$ and $v = 2A$, firms will have no incentive to deviate. ■

Our next result rules out situations in which firms: mix between advertising zero and both goods and mix between advertising one and both goods.

Proposition 2 *There exists no equilibrium in which $\lambda_0 + \lambda_2 = 1$ or $\lambda_1 + \lambda_2 = 1$.*

The proof is made up of the next two lemmas.

Lemma 5 *There exists no equilibrium in which firms mix between advertising no products and both products, i.e., $\lambda_0 + \lambda_2 = 1$.*

Proof. From lemma 1 we know that $p_{i0}^{NA} = v$. Now consider the pricing-strategy for the advertised products. First, notice that since $p_{i0}^{NA} = v$ we have $2p_{i0}^{NA} \geq p_{i2}^{A}$ (remember that p_{i2}^{A} is the *total* price of the advertised products).[4] Second, since consumers pay the monopoly-price for both products when a firm doesn't advertise, consumers will always prefer to visit a firm which advertises both products above a firm which doesn't advertise at all. Now suppose that an advertising firm charges p_{i2}^{A} with probability one, such that $p_{i2}^{A} = p_{A2}^{A} = p_{B2}^{A}$. We can derive the following profit-functions:

$$\pi_i(\lambda_{i0} = 1; p_{i0}^{NA} = v) = \lambda_0 v \tag{4.1}$$

$$\pi_i(\lambda_{i2} = 1; p_{i2}^{A}) = \lambda_0 p_{i2}^{A} + \frac{1}{2}\lambda_2 p_{i2}^{A} - \gamma A \tag{4.2}$$

Consider, for example, that firm A deviates by advertising two goods surely ($\lambda_{A2} = 1$) and offering a price slightly lower than p_{A2}^{A}, i.e., $p_{A2}^{A} = p_{i2}^{A} - \varepsilon$. For small ε we get: $\pi_A = \lambda_0 p_{i2}^{A} + \lambda_2 p_{i2}^{A} - \gamma A > \pi_i(\lambda_{i2} = 1; p_{i2}^{A}) = \lambda_0 p_{i2}^{A} + \frac{1}{2}\lambda_2 p_{i2}^{A} - \gamma A$. As a result, p_{i2}^{A} cannot be allocated a probability of one, since firms will have an incentive to undercut p_{i2}^{A}.[5]

[4]Notice that a consumer isn't really interested in the per-product prices of the goods, but is mainly interested in the total price of both goods. The reason for this is that consumers maximise the surplus of their whole shopping-list, since they can only visit one store.

[5]The observant reader might notice that $p_{i2}^{A} = 0$ cannot be undercut, but this pricing-strategy cannot be part of an equilibrium, because it leads to negative profits.

So, an advertising firm will mix in prices. As usual in this type of models, the price distribution must have no atoms and be defined on a connected support.[6] Let's denote this price distribution by the cumulative function: $F(p_{i2}^A)$ with support: $[\underline{p}_{i2}^A; \overline{p}_{i2}^A]$, such that we can write the profit-functions as follows:

$$\pi_i(\lambda_{i0} = 1; p_{i0}^{NA} = v) = \lambda_0 v \tag{4.3}$$

$$\pi_i(\lambda_{i2} = 1; p_{i2}^A) = \lambda_0 p_{i2}^A + \lambda_2(1 - F(p_{i2}^A))p_{i2}^A - \gamma A \tag{4.4}$$

Given the fact that $F(\overline{p}_{i2}^A) = 1$, we get that a firm advertising the upper bound $\overline{p}_{i2}^A$ obtains a profit of: $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A) = \lambda_{i0}\overline{p}_{i2}^A - \gamma A$. We can see that π_i is strictly increasing in $\overline{p}_{i2}^A$, therefore $\overline{p}_{i2}^A$ must be at its maximum: $2v$ (this is simply twice the monopoly-price, v). We can substitute $\overline{p}_{i2}^A = 2v$ and get equilibrium profits $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A) = 2\lambda_0 v - \gamma A$. Firms must be indifferent between playing $\lambda_{i0} = 1$ with $p_{i0}^{NA} = v$ and $\lambda_{i2} = 1$ with, for example a price $\overline{p}_{i2}^A = 2v$. Therefore, $\pi_i(\lambda_{i0} = 1; p_{i0}^{NA} = v) = \lambda_0 v = \pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A) = 2\lambda_0 v - \gamma A$. So, $\lambda_{i0} = \gamma A/v$ and the equilibrium profit must be equal to γA.

Now consider the following deviation: $\lambda_{i1} = 1$ with $p_{i1}^A = p_{i1}^{NA} = v$. Leading to profits of: $\pi_i(\lambda_{i1} = 1; p_{i1}^A = p_{i1}^{NA} = v) = 2\lambda_{i0}v - A = 2\gamma A - A$ (given $\lambda_0 = \gamma A/v$). We now have $2\gamma A - A > \gamma A$, which we can rewrite to: $\gamma A > A$. Clearly, the deviation is profitable and $\lambda_0 + \lambda_2 = 1$ can now be ruled out as part of an equilibrium. ■

Lemma 6 *There exists no equilibrium in which firms mix between advertising one product and both products,* $\lambda_1 + \lambda_2 = 1$.

Proof. Again, from Lemma 1 we know that $p_{i1}^{NA} = v$. First, let us consider the shopping-decision of the consumer. In the case that both firms advertise one good, consumers will simply visit the store with the lowest advertised price. Similarly, when both firms advertise both goods, consumers will visit the store with the lowest, advertised, total price (p_{i2}^A). In the case that one firm advertises one good and the other firm advertises both goods, consumers will go to the shop that gives them the highest surplus, so they will compare: $2v - v - p_{i1}^A$

[6]The proof is standard, see for example: Varian (1980).

with $2v - p_{i2}^A$.

Next, we argue that if $\lambda_1 + \lambda_2 = 1$ is part of an equilibrium, it must be the case that $p_{i2}^A \leq v$. The reasoning is as follows: for any $p_{i2}^A > v$, there exists a p_{i1}^A such that $p_{i1}^A + v < p_{i2}^A$. This implies that, for any $p_{i2}^A > v$, a firm can make a slightly better offer than p_{i2}^A, by advertising one good at a price of: $p_{i2}^A - v - \varepsilon$. For small ε we get: $\pi_i(\lambda_{i1} = 1; p_{i1}^A = p_{i2}^A - v - \varepsilon) > \pi_i(\lambda_{i2} = 1; p_{i2}^A)$ for any $p_{i2}^A > v$. The total price of the goods is for both strategies nearly equal, but advertising one good will give the firm at least the same expected demand as when it advertises both goods (the total price is slightly lower), while its advertising costs are lower. Clearly, any $p_{i2}^A > v$ is dominated by $p_{i1}^A = p_{i2}^A - v - \varepsilon$.

Now $p_{i2}^A \leq v$ implies that $2v - v - p_{i1}^A < 2v - p_{i2}^A$. So consumers will always prefer to visit a firm that advertised both goods over a firm that advertised only one good. We can derive: $\pi_i(\lambda_{i1} = 1; p_{i1}^A) = \frac{1}{2}\lambda_{i1}(p_{i1}^A + v) - A$. Clearly, firms have an incentive to slightly undercut any $p_{i1}^A > 0$; therefore, we'll get the Bertrand-result: $p_{i1}^A = 0$.

Let's consider the pricing-strategy for p_{i2}^A next: $\pi_i(\lambda_{i2} = 1; p_{i2}^A) = \lambda_{i1} p_{i2}^A + \frac{1}{2}\lambda_{i2} p_{i2}^A - \gamma A$. It is obvious that p_{i2}^A cannot be charged with probability one, because firms would have an incentive to undercut p_{i2}^A. So, again p_{i2}^A will be random, with price distribution: $F(p_{i2}^A)$, with support $[\underline{p}_{i2}^A; \overline{p}_{i2}^A]$. Again this price distribution must have no atoms and be defined on a connected support. We then have: $\pi_i(\lambda_{i2} = 1; p_{i2}^A) = \lambda_{i1} p_{i2}^A + \lambda_{i2}(1 - F(p_{i2}^A)) p_{i2}^A - \gamma A$. And: $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A) = \lambda_{i1} \overline{p}_{i2}^A - \gamma A$. We can see that π_i is strictly increasing in $\overline{p}_{i2}^A$, therefore $\overline{p}_{i2}^A$ must be at its maximum: v. Equilibrium requires that: $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A = v) = \lambda_{i1} v - \gamma A = \pi_i(\lambda_{i1} = 1; p_{i1}^A = 0) = \frac{1}{2}\lambda_{i0} v - A$. Solving gives: $\lambda_0 = 2A(\gamma - 1)/v$ and $\pi_i^* = \gamma A - 2A$. Only in the non-generic case of $\gamma = 2$, π_i^* is nonnegative. ■

Proposition 3 *There exist equilibria where* $\lambda_0 + \lambda_1 = 1$.

Proof. Recall from Lemma 1 that $p_{i0}^{NA} = p_{i1}^{NA} = v$. Given these prices, consumers will never prefer to visit a firm that didn't advertise at all rather than a firm that advertised one good. So, we must have: $\pi_i(\lambda_{i0} = 1; v) = \frac{1}{2}\lambda_0 v$ and $\pi_i(\lambda_{i1} = 1; v; p_{i1}^A) = \lambda_0(p_{i1}^A + v) + \frac{1}{2}\lambda_1(p_{i1}^A + v) - A$.

Now we'll argue that a firm advertising a price p_{1A}^A will mix in prices. First, notice that any price $p_{i1}^A > 0$, charged with probability one, cannot be part of an equilibrium. Firms have an incentive to undercut p_{i1}^A, but they cannot undercut $p_{i1}^A = 0$. We therefore consider $p_{i1}^A = 0$, substituting gives: $\pi_i(\lambda_{i1} = 1; v; 0) = \lambda_0 v + \frac{1}{2}\lambda_1 v - A$. Given $\lambda_0 + \lambda_1 = 1$ and $\pi_i(\lambda_{i0} = 1; v) = \pi_i(\lambda_{i1} = 1; v; 0)$ we can then derive: $\lambda_{i0} = 1 - 2A/v$ and $\pi_i^* = v - 2A$.

Now consider the following deviation: $\lambda_{i2} = 1$ with $p_{i2}^A = v$, resulting in profits of: $\pi_i(\lambda_{i2} = 1; p_{i2}^A = v) = v - \gamma A$. The deviation is profitable if: $v - 2A > v - \gamma A$, which always holds, except in the non-generic case of $\gamma = 2$. We have proven that p_{i1}^A must be random, p_{i1}^A will follow a price distribution $F(p_{i1}^A)$ with support $[\underline{p}_{i1}^A; \overline{p}_{i1}^A]$. We still have $\pi_i(\lambda_{i0} = 1; v) = \frac{1}{2}\lambda_0 v$, but now:

$$\pi_i(\lambda_{i1} = 1; v; p_{i1}^A) = \lambda_0(p_{i1}^A + v) + \lambda_1(1 - F(p_{i1}^A))(p_{i1}^A + v) - A \tag{4.5}$$

Given the fact that $F(\overline{p}_{i1}^A) = 1$, we have: $\pi_i(\lambda_{i1} = 1; v; \overline{p}_{i1}^A) = \lambda_{i0}(\overline{p}_{i1}^A + v) - A$. Clearly, π_i is strictly increasing in $\overline{p}_{i1}^A$, such that $\overline{p}_{i1}^A$ must be at its maximum: v. Now we can substitute $\overline{p}_{i1}^A = v$ and get: $\pi_i(\lambda_{i1} = 1; v; \overline{p}_{i1}^A) = 2\lambda_0 v - A = \pi_i(\lambda_{i0} = 1; v) = \frac{1}{2}\lambda_0 v$. We then derive $\lambda_0 = A/v$ and $\pi_i^* = A$. $\lambda_0 + \lambda_1 = 1$, therefore $\lambda_1 = 1 - A/v$. Next we have to check if $0 \leq \lambda_0 \leq 1$ and $0 \leq \lambda_1 \leq 1$. These conditions are satisfied if $v \geq A$, which holds by assumption. We can substitute λ_0 and λ_1 in (7) and given that profits must be equal to A, we can solve for $F(p_{i1}^A)$:

$$F(p_{i1}^A) = \frac{1}{1 - A/v} - \frac{2A}{(1 - A/v)(p_{i1}^A + v)} \tag{4.6}$$

Next, we have to find the lower bound of the distribution by setting $F(p_{i1}^A) = 0$. So, we'll have $\underline{p}_{i1}^A = 2A - v$. We have to be careful not to overlook the fact that $\underline{p}_{i1}^A \geq 0$, which gives us the following condition: $v \leq 2A$.

Now that we have defined the equilibrium, let us check for any possible deviations. First notice that firms will have no incentive to deviate by advertising both products. The only reason to advertise another product is to increase demand, but demand is already at its maximum when $\underline{p}_{i1}^A$ is played. For the same reason firms will have no incentive to deviate by setting a price $p_{i1}^A < \underline{p}_{i1}^A$. ■

Summary 1 There exists an equilibrium (EQ1) in which firms mix between advertising no products and one product. Moreover, firms mix over advertised prices, while the non-advertised price is constant at the monopoly-level. For this equilibrium to exist we need $A \leq v \leq 2A$.

We have defined an equilibrium where $\lambda_0 + \lambda_1 = 1$, but we are not done yet. What happens when the condition $A \leq v \leq 2A$ doesn't hold? We now show that there is another scenario in which the price distribution $F(p_{i1}^A)$ contains an atom at $p_{i1}^A = v$. Let's denote the probability that a firm plays the atom as λ_{1A} and the probability that a firm plays the distribution as λ_{1F}, such that: $\lambda_{1A} + \lambda_{1F} = \lambda_1$.

First, notice that $\pi_i(\lambda_{i0} = 1; v) = \frac{1}{2}\lambda_{i0}v$ has remained unchanged. We also still have that $\overline{p}_{i1}^A = v$. So, we'll get: $\pi_i(\lambda_{i1F} = 1; v; \overline{p}_{i1}^A) = 2\lambda_{i0}v - A$. Then we equate $\pi_i(\lambda_{i0} = 1; v) = \frac{1}{2}\lambda_{i0}$ to $\pi_i(\lambda_{i1F} = 1; v; \overline{p}_{i1}^A) = 2\lambda_{i0}v - A$ to find that $\lambda_{i0} = A/v$, $\pi_i^* = A$ and $\lambda_{i1} = 1 - A/v$. Notice that until now we still have the same results as with EQ1. Now consider the profit from playing the atom: $\pi_i(\lambda_{i1A} = 1; v; 0) = \frac{1}{2}\lambda_{i1A}v + (1 - \lambda_{i1A})v - A = \pi_i^* = A$. So, we have: $\lambda_{1A} = 2 - 4A/v$, then from $\lambda_0 + \lambda_{1A} + \lambda_{1F} = 1$ we can derive that $\lambda_{1F} = 3A/v - 1$. Next, we need that $0 \leq \lambda_0 \leq 1$, $0 \leq \lambda_{1A} \leq 1$ and $0 \leq \lambda_{1F} \leq 1$, which gives us the condition: $2A \leq v \leq 3A$.

It is now easy to derive $F(p_{i1}^A)$:

$$F(p_{i1}^A) = \frac{4Ap_{i1}^A/v + 2A - p_{i1}^A - v}{3Ap_{i1}^A/v + 3A - p_{i1}^A - v} \tag{4.7}$$

We then set $F(p_{i1}^A) = 0$ and find: $\underline{p}_{i1}^A = \frac{v-2A}{4A/v-1}$. Remember that $\underline{p}_{i1}^A \geq 0$, which holds if $v \geq 2A$.

Finally, we have to check for any possible deviations. First, consider that firms have no incentive to set a price such that p_{i1}^A: $0 < p_{i1}^A < \underline{p}_{i1}^A$. After all the expected demand from charging such a p_{i1}^A is equal to the expected demand from charging $\underline{p}_{i1}^A$. Another option, worth considering, is to deviate by advertising both products. A firm which deviates by playing: $\lambda_{i2} = 1$ with $p_{i2}^A = v$ would capture the entire market: $\pi_i(\lambda_{i2} = 1; v) = v - \gamma A$. To rule out this deviation we need $\pi_i^* = A \geq v - \gamma A$, which gives us the condition: $v \leq A(1+\gamma)$. We now check wether the conditions can be satisfied. We need $2A \leq v \leq A(1+\gamma)$, which is sustainable.

Summary 2 There exists an equilibrium (EQ2) (in essence similar to EQ1) in which firms mix between advertising no products and one product. Moreover, firms mix the advertised prices over a certain price distribution and the perfect competitive price, while the non-advertised price is constant at the monopoly-level. This equilibrium exists if $2A \leq v \leq A(1+\gamma)$.

Proposition 4 *There exists an equilibrium where* $\lambda_0 + \lambda_1 + \lambda_2 = 1$.

Proof. First, we'll argue that a consumer will prefer to visit the firm which advertised the most products. For this we need that: $2p_{i0}^{NA} \geq p_{i1}^{A} + p_{i1}^{NA} \geq p_{i2}^{A}$, which we can rewrite, given Lemma 1, as: $2v \geq p_{i1}^{A} + v \geq p_{i2}^{A}$. Firms will not set a price above v, so it is clear that $2v \geq p_{i1}^{A} + v$. Next, recall from Lemma 6 that $p_{i2}^{A} \leq v$, such that $p_{i1}^{A} + v \geq p_{i2}^{A}$ (recall that any $p_{i2}^{A} > v$ is dominated by $p_{i1}^{A} = p_{i2}^{A} - v - \varepsilon$).

Now we can determine the equilibrium profits for the different action. $\pi_i(\lambda_{i0} = 1) = \lambda_0 v$, $\pi_i(\lambda_{i1} = 1; p_{i1}^{A}) = \lambda_0(p_{i1}^{A}+v)+\frac{1}{2}\lambda_1(p_{i1}^{A}+v)-A$ and $\pi_i(\lambda_{i2} = 1; p_{i2}^{A}) = (\lambda_0+\lambda_1)p_{i2}^{A}+\frac{1}{2}\lambda_2 p_{i2}^{A}-\gamma A$. We can see that firms have an incentive to (slightly) undercut p_{i1}^{A} and p_{i2}^{A}. Although, $p_{i2}^{A} = 0$ can not be undercut, it can't be part of an equilibrium-pricing strategy, because it would lead to negative profits. Therefore, we must have that p_{i2}^{A} is random. Let us denote $G(p_{i2}^{A})$, with support $[\underline{p}_{i2}^{A}; \overline{p}_{i2}^{A}]$, as the cumulative price distribution function of p_{i2}^{A}. We argue that $\overline{p}_{i2}^{A} = v$: $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^{A}) = (\lambda_0 + \lambda_1)\overline{p}_{i2}^{A} - \gamma A$; π_i is strictly increasing in $\overline{p}_{i2}^{A}$ and we know that $\overline{p}_{i2}^{A} \leq v$. Since $p_{i1}^{A} = 0$ doesn't necessarily lead to negative profits, we have a number of options for the pricing strategy for p_{i1}^{A}: (i) firms can charge $p_{i1}^{A} = 0$ with certainty, (ii) they can play a price distribution or (iii) they can mix between $p_{i1}^{A} = 0$ and a price distribution (like we observed in EQ2).

Let us consider (ii), i.e. suppose that firms play a price distribution $F(p_{i1}^{A})$ with a lower bound: $\underline{p}_{i1}^{A} > 0$ ($\underline{p}_{i1}^{A} = 0$ falls under the last option). Firms now have an incentive to deviate by charging a $p_{i2}^{A} = \underline{p}_{i1}^{A} + v \geq v$, because $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^{A} = v) = (\lambda_0 + \lambda_1)v - \gamma A < \pi_i(\lambda_{i2} = 1; p_{i2}^{A} = \underline{p}_{i1}^{A} + v) = (\lambda_0 + \lambda_1)(\underline{p}_{i1}^{A} + v) - \gamma A$. We can discard the second option.

Let's suppose that firms play the first option, that is they charge $p_{i1}^{A} = 0$ with certainty (if they advertise one good). We must have: $\pi_i(\lambda_{i0} = 1) = \lambda_0 v = \pi_i(\lambda_{i1} = 1; p_{i1}^{A} = 0) = \lambda_0 v + \frac{1}{2}\lambda_1 v - A$. We can derive: $\lambda_1 = 2A/v$. Next, we also must have that: $\pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^{A} = v) = (\lambda_0 + \lambda_1)v - \gamma A = \lambda_0 v$, from which can derive: $\lambda_1 = \gamma A/v$. Clearly, $\gamma \leq 2$, such that we can discard the first option, since it can only lead to a non-generic result.

We consider the last option: firms charge $p_{i1}^A = 0$ with probability λ_{i1A} and they play the price distribution, $F(p_{i1}^A)$, with probability λ_{i1F}, with $\lambda_{i1A} + \lambda_{i1F} = \lambda_{i1}$. It is easy to find that $\overline{p}_{i1}^A = v$. We then get: $\pi_i(\lambda_{i0} = 1) = \lambda_0 v = \pi_i(\lambda_{i1F} = 1; \overline{p}_{i1}^A = v) = 2\lambda_0 v - A$, from which we can derive: $\lambda_0 = A/v$ and $\pi_i^* = A$. Next, observe that: $\pi_i(\lambda_{i1A} = 1; p_{i1}^A = 0) = (\lambda_0 + \lambda_{1F})v + \frac{1}{2}\lambda_{i1A}v - A = \pi_i(\lambda_{i2} = 1; \overline{p}_{i2}^A = v) = (\lambda_0 + \lambda_1)v - \gamma A$, we get: $\lambda_{i1A} = 2A(\gamma - 1)/v$, $\lambda_{1F} = A(2-\gamma)/v$ and $\lambda_2 = 1 - A(1+\gamma)/v$. We are now ready to find the price distributions:

$$F(p_{i1}^A) = \frac{Ap_{i1}^A(3-\gamma)/v + A(1-\gamma)}{Ap_{i1}^A(2-\gamma)/v + A(2-\gamma)} \tag{4.8}$$

$$G(p_{i2}^A) = \frac{p_{i2}^A - A(1+\gamma)}{p_{i2}^A - Ap_{i2}^A(1+\gamma)/v} \tag{4.9}$$

Then we set $F(p_{i1}^A)$ and $G(p_{i2}^A) = 0$ to find: $\underline{p}_{i1}^A = \frac{v(\gamma-1)}{3-\gamma}$ and $\underline{p}_{i2}^A = A(1+\gamma)$. In equilibrium we must have that λ_0, λ_{i1A}, λ_{1F}, λ_2, $\underline{p}_{i1}^A$ and $\underline{p}_{i2}^A$ are all nonnegative, from which we get the condition: $v \geq A(1+\gamma)$. ■

Summary 3 There exists an equilibrium (EQ3) in which firms mix between advertising 0, 1 and 2 goods. The advertised prices are random, with some positive probability that firms charge the perfect-competitive price. Non-advertised prices are constant at the monopoly-level. This equilibrium exists if $v \geq A(1+\gamma)$.

Figure 4.1 illustrates that there is a unique equilibrium for any given set of values of the parameters. The comparative statics are similar for all the equilibria we have characterized. The advertising intensity of firms decreases in advertising costs: firms advertise less often and if they advertise, they advertise a lower number of goods with an increased probability. A decrease in advertising intensity leads to higher advertised prices, such that advertised prices are increasing in advertising costs. When advertising costs increase, the equilibrium changes continuously from the perfectly competitive outcome (Bertrand result) to the monopoly outcome (Diamond result). Profits are increasing in advertising costs; consumers rely on advertising to become aware of prices, such that firms only compete through advertising.

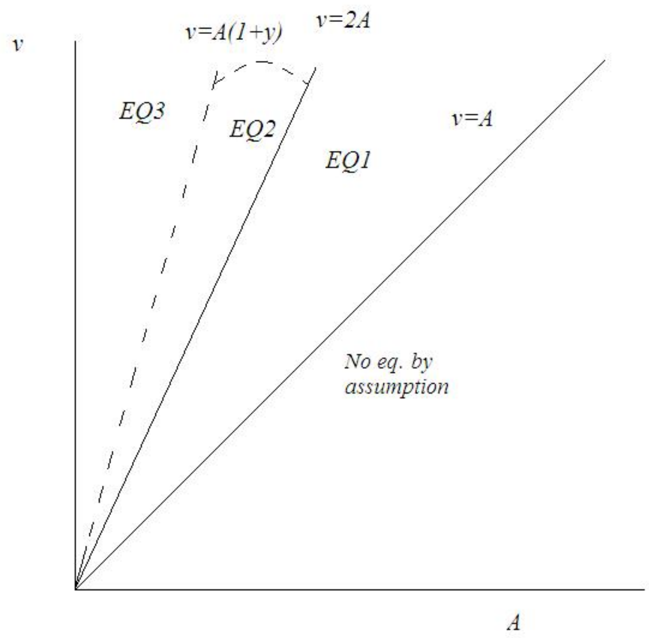

Figure 4.1: Existence of equilibria

When advertising costs are sufficiently high, firms do not advertise at all and are able to charge the monopoly price for both goods.

Notice that the model is not really suited for welfare analysis. Our assumption that demand is inelastic, leads to the result that advertising is wasteful. The reason behind this result is that a reduction in prices (through the competition effect of advertising) does not increase welfare, because there is no dead-weight loss when demand is inelastic.

Chapter 5

Extension

Until now we have assumed that consumers place equal value on both goods. We have seen that this leads to the result that firms do not care about which of the two goods they advertise (if they advertise one good). In this chapter we analyze if firms remain indifferent or whether they'll have some preference, for one good or the other, when consumers do not value the goods equally. This is particularly interesting, because the empirical literature suggests that supermarkets tend to promote branded-goods rather than own brand-goods. We will make a slight modification to the basic model: let us assume that consumers now have a reservation price of v_H for one good (good H) and a reservation price of v_L for the other good (good L), with $v_H > v_L$. At first we will restrict our analysis to the case where a firm advertises one good (a potential outcome of its mixed strategy), because only in this case a firm has to make a decision which of the two goods it will advertise.[1] This analysis will be helpful to interpret the equilibrium that we will characterize later on.

First, we need to introduce some new notation. Let us denote p_H^{NA} as the non-advertised price a firm charges for good H and p_L^{NA} as the non-advertised price a firm charges for good L. Let p_H^A then denote the advertised price a firm charges for good H and let p_L^A denote the advertised price a firm charges for good L; finally let p_1^T denote the total price of good H and good L when a firm advertises one good. Notice that firms will still charge the monopoly-price for the non-advertised goods: $p_H^{NA} = v_H$ and $p_L^{NA} = v_L$. After all, consumers will still

[1] In fact for certain deviations from the equilibria, firms also have to decide which good to advertise. However, we are only interested in which good firms prefer to advertise in equilibrium at this point.

visit one firm at maximum.

Now consider which of the two goods a firm would rather advertise. One might expect that a firm would rather advertise good L, such that it can make a large profit on good H ($p_H^{NA} = v_H > p_L^{NA} = v_L$). This is, however, not the case. Consumers are indifferent between visiting a firm which advertises good L and good H when: $p_H^{NA} + p_L^A = p_L^{NA} + p_H^A$. Given $p_H^{NA} = v_H$ and $p_L^{NA} = v_L$, we have: $v_H + p_L^A = v_L + p_H^A$. Therefore, firms are indifferent between advertising good H at p_H^A and advertising good L at $p_L^A = p_H^A + v_L - v_H$. The intuition behind this result is quite straightforward: since consumers will visit one firm maximum, only the sum of the prices of good H and good L matters. Suppose a firm advertises good L. If this firm would advertise good H instead, its non-advertised price would drop from $p_H^{NA} = v_H$ to $p_L^{NA} = v_L$. The firm can make up for this reduction by setting a relatively high price for the good it advertises ($p_L^A < p_H^A$).

A firm is, however, not able to set a $p_L^A = p_H^A + v_L - v_H$ for all p_H^A, because, for small p_H^A, p_L^A will become negative. Therefore, if firms want to advertise one good and set a relatively low *total* price, then they will advertise good H. Advertising serves as a commitment-device and goods with a relative high value can guarantee consumers a higher surplus than goods with a relative low value.[2]

To illustrate these results we next characterize an equilibrium in which firms mix between advertising one good and no good at all.

Proposition 5 *If $v_L \leq 2A \leq v_H + v_L$, then the following strategy constitutes an equilibrium: firms do not advertise at all with probability $\lambda_0 = 2A/(v_H + v_L)$ and they advertise one good with probability $\lambda_1 = 1 - 2A/(v_H + v_L)$. The non-advertised prices are set at the monopoly-level: $p_H^{NA} = v_H$ and $p_L^{NA} = v_L$. The total price for the basket of goods H and L when a firm advertises one good (p_1^T) follows a price distribution $F(p_1^T)$ with support: $[2A; v_H + v_L]$.*

$$F(p_1^T) = \frac{1}{\lambda_1} - \frac{2A}{\lambda_1 p_1^T} \tag{5.1}$$

For any $p_1^T \geq v_H$ firms are indifferent between advertising good H at p_H^A and good L at

[2]Advertising good H can guarantee a maximum surplus of v_H while advertising good L can only guarantee a maximum surplus of v_L.

$p_L^A = p_H^A + v_L - v_H$, *while for any* $p_1^T < v_H$ *firms prefer to advertise good H.*

Proof. The main part follows the proof of Proposition 3 very closely.[3] From the analysis above it follows that firms are indifferent between advertising good H at p_H^A and good L at $p_L^A = p_H^A + v_L - v_H$. Notice that $p_1^T = p_L^A + v_H = p_H^A + v_L$ if $p_L^A = p_H^A + v_L - v_H$. We can then write: $p_L^A = p_1^T - v_H$. Since prices cannot be negative, we must have that: $p_L^A = p_1^T - v_H \geq 0$. This implies that firms are indifferent between advertising good H and good L when $p_1^T \geq v_H$ and prefer to advertise good H when $p_1^T < v_H$. ■

We can find the probability that a firm prefers to advertise good H when it advertises one good by setting $p_1^T = v_H$ in the distribution function: $(v_H - 2A)/(\lambda_1 v_H)$. Clearly, firms may only prefer to advertise good H in equilibrium when the advertising costs are sufficiently low: $v_H \geq 2A$. Also notice that the probability that firms prefer to advertise good H decreases in advertising costs. The intuition behind this result is that when advertising costs increase firms compete less heavily in advertising. This leads to an increase in the (average) prices. We have seen that firms only prefer to advertise good H when they want to charge a relatively low total price (p_1^T).

Our results differ from Lal & Matutes where there are two equilibria in the case of different reservation prices: in one equilibrium both firms advertise good L and in the other equilibrium both firms advertise good H. There is no difference between these equilibria regarding existence conditions and expected profits. Therefore, firms do not prefer to advertise one good over the other.

The reason why firms never prefer to advertise good H over advertising good L in Lal & Matutes is that they put no restriction on prices. While in our model negative prices are not allowed, these prices are allowed in their model. If, however, Lal & Matutes had a restriction on prices then the equilibrium in which both firms advertise good H would hold for a larger region of the parameters than the equilibrium in which both firms advertise good L. It is then clear that putting a restriction on prices has a major impact on the results, because if prices can be negative it is no longer the case that advertising a good with a relatively high value guarantees consumers a higher surplus than advertising a good with a relatively low

[3] In fact if we set $v_H = v_L = v$, we get EQ1 of the basic model.

value. For both Lal & Matutes and our model, it applies that only when there is a restriction on prices (not necessarily as strict as non-negativity of prices) firms may prefer to advertise good H over good L. Even when we consider goods for which consumer have a relatively inelastic demand, it is reasonable to assume that when prices are negative enough demand will increase severely. We therefore argue that putting a restriction on prices is more in line with reality than putting no restriction on prices.

Lal & Matutes also argue that only goods with a relatively low value will be used as loss-leaders. This result can lead to some confusion, because it is **not** the case that it is more likely that firms advertise good H than good L. The idea behind their result is that when both firms advertise good L the advertised price is lower than if both firms advertised good H. Because unit-selling costs are equal for good L and good H, it is more likely that the advertised price of good L is below unit-selling costs than the advertised price of good H. Their result is actually in line with our results. Our results imply that firms are indifferent between advertising good L at a relatively low price and advertising good H at a relatively high price. When unit-selling costs are equal for both goods then it is indeed more likely that good L is priced below unit-selling costs than good H. Like Lal & Matutes we find that goods with a relatively low value are more likely to be loss-leaders, we also find that it is not more likely that these goods are advertised relative to goods with a high value.

Summary 4 *A firm will never prefer to advertise the good with the lower reservation price. However, a firm prefers to advertise the good with the higher reservation price when it wants to set a highly competitive **total** price.*

Chapter 6

Discussion

In this chapter we discuss the expected implications of relaxing two of our assumptions, namely: (i) consumers visit one firm at maximum and (ii) demand is inelastic. These assumptions help to illustrate the main ideas of this paper, but they lead to three unsatisfying results. First, firms are indifferent with respect to which of the two goods they advertise.[1] Secondly, firms are indifferent with respect to the individual prices of goods when they advertise both goods. And last firms always set the monopoly price for unadvertised goods. Next, we argue that these results are intimately related to assumptions (i) and (ii).

Suppose that consumers do not visit one firm at maximum, but rather have the option to visit a second firm upon paying a certain search cost. Firms now run the risk that consumers will 'hunt for bargains' when one firm advertises good A and the other firm advertises good B. When search costs are sufficiently low consumers will visit both stores and only buy the 'cheap' advertised goods, leaving both firms with relatively low sales. This is not a problem if, in equilibrium, firms advertise the same good, however, our analysis indicates that such an equilibrium does not exist. We therefore suspect that, in equilibrium, firms will choose their advertised good according to a mixed strategy, but this conjecture has to be confirmed by concrete analysis. This would imply that firms no longer choose the advertised good completely at random as is the case in our current model.

[1] The problem with this result is that there are infinitively many equilibria. When we relax the assumptions (i) and/or (ii) we expect to find a unique equilibrium in mixed strategies.

Another option for firms to reduce the risk of 'bargain-hunting' is to minimize the difference between the prices of their individual goods. For example, when a firm advertises both goods it rather sets two moderate prices for both goods than a low price for one good and a high price for the other. This way the firm can prevent a consumer from only buying the good with a low price. This implies that firms are no longer indifferent with respect to the individual prices of goods when they advertise both goods, but rather offer identical discounts on both goods. One can also imagine that firms no longer always charge the monopoly price for unadvertised goods, but rather set a price which reduces the probability that consumers continue their search.

In search models with only one good firms usually set prices (in equilibrium) in such a way that consumers do not continue their search after they entered the first store.[2] Firms do not have an incentive to deviate from this equilibrium by raising their price, because consumers would then continue their search, leaving the firm with no sales at all. It is uncertain if firms adopt a similar same strategy in an advertising and search model with multiple goods. If firms indeed use a mixed strategy to determine which good they advertise, then firms are uncertain which good its rival advertises. There is a probability that firms advertise the same good and a probability that firms advertise different goods. Suppose that the firms charge prices in such a way that consumers never shop around. When a firm deviates by charging a slightly higher non-advertised price, then it is not necessarily the case that consumers continue their search after entering this firms store. Consumers will not continue their search if the firms advertise the same good; the increase in the non-advertised price is smaller than the extra costs of visiting another firm. On the other hand, if firms advertise different goods, than this small increase in the non-advertised price can be just sufficient for consumers to induce them to shop around and buy the relatively cheap advertised good from both firms. Clearly, the effect on demand, when a firm deviates by raising its non-advertised price, is less severe than in the search model with one good. It is therefore unclear if an equilibrium in which firms set their prices in such a way that consumers never shop around is sustainable. We suspect that there exists an equilibrium in which firms do not charge low prices to prevent consumers from shopping around completely, but rather set moderate

[2]See for example Stahl (1989).

prices and risk that consumers may shop around (depending on whether firms happen to advertise the same or different goods).

Another important assumption has been that consumers hold inelastic demands. Suppose now that demand were elastic. In that case firms would no longer be indifferent with respect to individual prices, because demand would now depend on the individual prices rather than on the sum of the individual prices. Depending on the shape of the demand-function, firms might prefer to set moderate prices for both goods over setting a high price for one good and a low price for the other good. The main incentive for firms to advertise both goods rather one is that this increases their ability to guarantee the consumers a certain amount of surplus. With elastic demand firms have an additional incentive to advertise both goods: it allows them more flexibility in setting their prices.

Suppose a firm advertises one good at a relatively low price. Since it must set the monopoly price for the non-advertised good, the firm might do better by advertising both goods and setting a lower price for the good it didn't advertise and a higher price for the good it did advertise. The firm can keep the amount of surplus it guarantees the consumers constant, while (potentially) increasing its sales. While this deserves a separate study, we constructed some scenario's in which this was indeed the case.

Another implication of relaxing the assumption of inelastic demand is that firms are no longer always indifferent with respect to which good they advertise. In the extension of the model we saw that goods with a relatively high value are attractive to advertise, because they give firms the highest capacity to commit to low prices. With elastic demand, however, there is another product characteristic which determines which good is attractive to advertise, namely, the elasticity of demand. Firms prefer to advertise goods which are able to guarantee consumers a high surplus relative to the amount of sales sacrificed to guarantee this surplus, which basically depends on the slope of the demand curve. We therefore believe that the preference of firms to advertise some good and not others depends, among possibly others, on: (i) the relative value of the good and (ii) the elasticity of demand.

Chapter 7

Conclusion

In this paper we try to explain why advertising and pricing strategies in retailing sometimes appear to be random. We develop a stylized advertising model in which two firms compete for the demand of two independent goods (neither substitutes nor complements) through advertising. Consumers are unaware of prices unless advertised and choose to visit one store (convenience shopping) based on expected prices. When consumers enter a store they learn the prices of the non-advertised goods of that firm and decide which goods to purchase. We show that firms have monopoly power over the consumers who enter their store, such that firms always charge the monopoly price for non-advertised goods. Furthermore, we show that firms use a mixed strategy to determine their advertising intensity and the prices of the goods they advertise. The intuition behind this result is that firms cannot advertise with certainty in equilibrium, because this leads to Bertrand competition, such that the prices are to low to cover the advertising costs. Our model is able to explain: why firms advertise bread this week and beer in the next; why advertised prices are dispersed; why non-advertised prices are much less volatile; why firms advertise more heavily in one week than in the other.

We confirm the result of Lal & Matutes that advertising serves as a commitment-device and argue that the main reason for firms to increase advertising intensity is to be able to increase their capacity to commit to low prices. The idea behind this result is that advertising one good can only guarantee consumers a certain maximum amount of surplus (namely the reservation price), while advertising two goods can potentially guarantee consumers a higher amount of surplus (namely the sum of the reservation prices). Since advertising is costly, firms only advertise both goods when they actually want to use their relative high capacity to commit by setting a low total price (the sum of the prices of both goods). This implies that the higher the advertising intensity of a firm, the lower the sum of the prices of the goods of that firm will be. Therefore, consumers prefer to visit the firm with the highest advertising intensity, such that advertising actually increases store traffic.

When advertising costs decrease, the equilibrium changes continuously from the monopoly outcome to the perfectly competitive outcome, such that, in principle, any (positive) price smaller or equal than the reservation price can be observed in an advertisement, while non-advertised prices are always set at the monopoly level.

In the extension chapter we have relaxed the assumption that the reservation prices are equal and showed that firms never prefer to advertise a good with a low reservation price, but may prefer to advertise a good with a high reservation price. The idea behind this result is that goods with a relatively high value can guarantee consumers a higher surplus than goods with a relatively low value. This result crucially depends on the assumption that prices are restricted to a minimum, which is, as we have argued earlier, a reasonable assumption. Lal & Matutes did not put a restriction on prices, which explains their result that firms never prefer to advertise the good with a high reservation price. In our model it is also true that whenever a firm sells a good below unit selling costs (loss-leader pricing), it is more likely that this good has a low reservation price than a high reservation price.

In the Discussion chapter we identified three unsatisfying results of our model: (i) firms are indifferent with respect to which of the two goods they advertise, (ii) firms are indifferent with respect to the individual prices of goods when they advertise both goods and (iii) firms always set the monopoly price for unadvertised goods. We argued that when we relax the assumptions that demand is inelastic and that consumers visit one firm at maximum, these results will no longer be an issue. Especially, studying elastic demand can provide us with new insights. For one, we suspect that both the relative value of goods and their elasticities of demand are important factors in determining which goods firms prefer to advertise. It also sheds light on another reason for firms to increase their advertising intensity besides the increase in the ability of firms to use advertising as a commitment-device , namely increasing their advertising intensity allows firms more flexibility in their pricing-decision. Therefore, it is desirable to concentrate future research on relaxing the assumptions we mentioned above. Furthermore, our analysis concentrates on a model in which the goods are independent. The model could be modified to study substitutes and complements.

Appendix A

Positive unit selling costs

In the appendix we give a summary of our results when we relax the assumption that unit selling costs are equal to zero. Assume that unit selling costs are equal to $c \geq 0$ and that $v \geq A + c$.

The equilibria are quite similar to the equilibria of the basic model:
Eq.1: $\pi^* = A$, $\lambda_0 = \frac{A}{v-c}$, $\lambda_1 = 1 - \frac{A}{v-c}$ and $p^{NA} = v$.

$$F(p_{i1}^A) = \frac{1}{\lambda_1} - \frac{2A}{\lambda_0(p_{i1}^A + v - 2c)} \tag{A.1}$$

$\sigma = [2A + 2c - v; v]$. This equilibrium exists if $A + c \leq v \leq 2A + 2c$. Firms can charge a price below marginal costs if: $2A + 2c - v < c$, which we can rewrite to: $v > 2A + c$.

Eq.2: $\pi^* = A$, $\lambda_0 = \frac{A}{v-c}$, $\lambda_1 = 1 - \frac{A}{v-c}$, $\lambda_{1F} = \frac{4A-v+2c}{v-2c}$, $\lambda_{1A} = 1 - \lambda_{1F}$, $p_A^A = 0$ and $p^{NA} = v$.

$$F(p_{i1}^A) = 1 + \frac{\lambda_0}{\lambda_{1F}} - \frac{2A}{\lambda_{1F}(p_{i1}^A + v - 2c)} \tag{A.2}$$

$\sigma = [\frac{2A}{\lambda_0+\lambda_{1F}} + 2c - v; v]$. This equilibrium exists if $2A + 2c \leq v \leq A(\gamma + 1) + 2c$. Firms can charge a price below marginal costs if $c > 0$.

Eq.3: $\pi^* = A$, $\lambda_0 = \frac{A}{v-c}$, $\lambda_{1F} = \frac{A(3-\gamma)}{v-2c} - \lambda_0$, $\lambda_{1A} = \frac{2A(\gamma-1)}{v-2c}$, $\lambda_2 = 1 - \frac{A(\gamma-1)}{v-2c}$, $p_A^A = 0$ and $p^{NA} = v$.

$$F(p_{i1}^A) = 1 + \frac{\lambda_0}{\lambda_{1F}} - \frac{2A}{\lambda_{1F}(p_{i1}^A + v - 2c)} \tag{A.3}$$

$$G(p_{i2}^A) = \frac{1}{\lambda_2} - \frac{A(\gamma+1)}{\lambda_2(p_{i1}^A - 2c)} \tag{A.4}$$

$\sigma = [A(\gamma+1) + 2c; v]$. This equilibrium exists if $v \geq A(\gamma+1) + 2c$. Firms can charge a price below marginal costs if $c > 0$. Notice that we have defined an unique equilibrium for all possible values of the parameters.

Summary 5 *We can observe loss-leader pricing when $v > 2A + c$.*

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